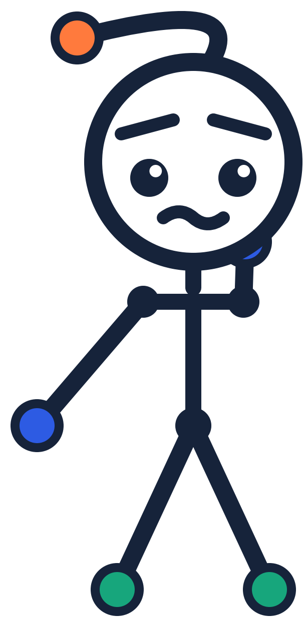
Young Innovators
FOR CHANGE
innovateyouth.org

A FULL-YEAR ENRICHMENT PROGRAM · GRADE 5

The Best Way

36 weekly assignments on how much a method costs, and how to tell which one is *better*



This book belongs to

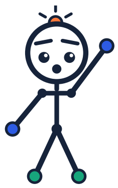
____

Teacher ______________________________ School year ______________________________

One assignment a week · two pages, front and back · about 30 to 40 minutes
Built to sit alongside the California Grade 5 math curriculum, not to replace it.

How This Book Works

Read this page with a grown-up before Week 1.



Hello again. I am Pip.

Grade 4 asked whether a thing is possible. This year asks what it **costs**. Two methods can both get the right answer and one can do ten thousand times more work, and by Week 34 you will be able to say exactly how much more.

One assignment a week, two pages

Page 1 has a note about what you are doing in class this week, a two-minute **Number Warm-Up**, the **Words to keep**, and the main problem.

Page 2 has more to try, then **Add to your Method Book**, a **Talk About It** question, a hint, and a ★ **Challenge Zone**.

The Method Book

One folder, built all year, one page per method. Every page needs the method **and** what it costs, never just the method. By Week 36 it is about thirty-three pages and it is the part you keep.

Three rules for this book

1. Say what a **step** is before you count them. Comparisons, boxes and days are three different currencies and they do not mix.
2. A method is only half an answer. The other half is a reason nothing can beat it.
3. Faster is not always better. Sometimes the simpler method is the right one, and Week 35 is about how to tell.

For the grown-up helping

The question to keep asking is **“how much would that cost if the list were a thousand times longer?”** Children arrive able to say one method feels faster. The year is about replacing that feeling with a number they can defend. When a child says a method is better, ask them better by how much, and in what units.

The Year at a Glance

Weeks 1–18. The last column is what the class is usually doing that week.

WK	ASSIGNMENT	STRAND	IN CLASS THIS WEEK
TRIMESTER 1 · WEEKS 1–12			
1	**What Is a Step?**	Steps & Cost	Multi-digit multiplication and division (5.NBT.5, 5.NBT.6)
2	**Two Sorts, Counted**	Steps & Cost	Multi-digit operations; expressions (5.NBT.5, 5.OA.1)
3	**Sorting by Splitting**	Steps & Cost	Problem solving with whole numbers (5.OA.1, 5.NBT.5)
4	**Searching a Sorted List**	Steps & Cost	Place value and powers of ten (5.NBT.1, 5.NBT.2)
5	**Doubling and Halving**	Number Structure	Powers of ten and place value (5.NBT.2)
6	**The Greedy Rule**	Steps & Cost	Decimal operations with money (5.NBT.7)
7	**When Greedy Fails**	Steps & Cost	Decimal and whole number operations (5.NBT.7, 5.OA.1)
8	**Shortest Path With Costs**	Dots & Lines	Adding and subtracting decimals (5.NBT.7)
9	**The Cheapest Network**	Dots & Lines	Adding decimals; problem solving (5.NBT.7, 5.OA.1)
10	**Packing the Boxes**	Steps & Cost	Volume and measurement (5.MD.3, 5.MD.5)
11	**Scheduling the Jobs**	Steps & Cost	Problem solving; measurement (5.OA.1, 5.MD.1)
12	**Method Fair**	Steps & Cost	Problem solving (5.OA.1)
TRIMESTER 2 · WEEKS 13–24			
13	**Case Files**	Logic & Proof	Review & reasoning
14	**Counting on a Grid**	Ways of Counting	The coordinate plane (5.G.1, 5.G.2)
15	**Paths That Avoid a Block**	Ways of Counting	The coordinate plane; problem solving (5.G.1, 5.OA.1)
16	**Two Circles, Then Three**	Ways of Counting	Problem solving with whole numbers (5.OA.1)
17	**Counting the Leftovers**	Ways of Counting	Place value and multi-digit numbers (5.NBT.1, 5.OA.1)
18	**Sharing Fairly**	Fractions	Dividing with unit fractions (5.NF.7)

The Year at a Glance

Weeks 19–36. The last column is what the class is usually doing that week.

WK	ASSIGNMENT	STRAND	IN CLASS THIS WEEK
19	**A Fraction of a Fraction**	Fractions	Multiplying fractions as area (5.NF.4, 5.NF.6)
20	**Powers and Place Value**	Number Structure	Powers of ten and place value (5.NBT.2)
21	**Base Five**	Number Structure	Place value and powers (5.NBT.1, 5.NBT.2)
22	**Carrying in Base Five**	Number Structure	Adding within a place value system (5.NBT.7, 5.NBT.1)
23	**How a Machine Adds**	Steps & Cost	Place value in another base (5.NBT.1)
24	**Logic Gates**	Steps & Cost	Reasoning and patterns (5.OA.1, 5.OA.3)
TRIMESTER 3 · WEEKS 25–36			
25	**Puzzle Fair**	Logic & Proof	Review & reasoning
26	**Flow Through a Network**	Dots & Lines	Problem solving; volume (5.OA.1, 5.MD.5)
27	**The Bottleneck**	Dots & Lines	Problem solving (5.OA.1)
28	**Matching People to Jobs**	Dots & Lines	Problem solving (5.OA.1)
29	**Coloring, and How Good Is Greedy?**	Dots & Lines	Problem solving (5.OA.1)
30	**A Lower Bound**	Logic & Proof	Problem solving and reasoning (5.OA.1)
31	**Rules That Look Back**	Ways of Counting	Patterns and rules (5.OA.3)
32	**A Rule From a Table**	Ways of Counting	Generating patterns and graphing pairs (5.OA.3, 5.G.1)
33	**Two Rules, One Answer**	Logic & Proof	Reasoning about patterns (5.OA.3, 5.OA.1)
34	**Sorting Really Big Lists**	Steps & Cost	Multi-digit operations; patterns (5.NBT.5, 5.OA.3)
35	**Which Method Would You Pay For?**	Steps & Cost	Decimal operations in context (5.NBT.7, 5.OA.1)
36	**The Method Book, Finished**	Steps & Cost	Review of the year

WEEK 1 STEPS & COST

What Is a Step?

Before you can say one method is better, you have to say what you are counting.

IN CLASS THIS WEEK
Multi-digit multiplication and division (5.NBT.5, 5.NBT.6)

NUMBER WARM-UP · 2 MINUTES
Which is more work: adding two numbers, or comparing two numbers?

WORDS TO KEEP

step one unit of work. In this book a step is almost always **one comparison**: looking at two things to see which is bigger

cost how many steps a method takes. Not how long it feels

Eight cards face up. Find the biggest, and count how many times you compare two cards.

the cards								
	7	3	9	1	5	8	2	6

1 The obvious method.

Hold the best you have seen so far. Compare it with the next card. If the next card wins, it becomes the new best.

a. You start by holding the first card. How many cards are left to compare it with? ________

b. So the cost is ________ comparisons.

2 Could anybody do better?

a. Every card that is **not** the biggest has to lose at least one comparison, or you could not know it lost. How many losers are there? ________

b. Each comparison produces exactly ________ loser.

c. So **no** method can do it in fewer than ________ comparisons. Yours is already the best there is.

3 Bigger piles.

Cards	comparisons to find the biggest
8	7
20	
100	
n	

✎ ADD TO YOUR METHOD BOOK

Start the Method Book. Page 1: **finding the biggest of n things costs n − 1 comparisons, and nothing can beat that.** Write the loser argument underneath, because it is the part that makes the claim worth anything.

💬 TALK ABOUT IT

You showed the method is best possible without inventing any other methods. How?

Why is counting comparisons fairer than timing yourself with a stopwatch?

STUCK? TRY THIS

Count the **losers**, not the winners. Every comparison knocks out exactly one card, and everything except the champion has to be knocked out.

★ CHALLENGE ZONE

Now find the biggest **and** the second biggest. The lazy way is to find the biggest, put it aside, and start again. How many comparisons is that for 8 cards?

There is a better way. Play a knockout tournament. The second biggest must have lost to the winner, so only the winner's victims need checking. How many comparisons now?

WEEK 2 STEPS & COST

Two Sorts, Counted

Two methods, the same cards, the same finish, and very different bills.

IN CLASS THIS WEEK
Multi-digit operations; problem solving (5.NBT.5, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
What is 8 × 7? Now halve it.

WORDS TO KEEP

selection sort find the smallest, put it first, then find the smallest of the rest

insertion sort pick up each card and slide it left into place

The same eight cards, twice. Count comparisons both times.

start								
	7	3	9	1	5	8	2	6

1 Selection sort first.

To find the smallest of 8 you need 7 comparisons. To find the smallest of the remaining 7 you need 6. And so on.

a. Write the run: 7 + 6 + 5 + 4 + 3 + 2 + 1 = ________

b. Does that total depend on how the cards started? ________ Try it on cards that are already sorted.

2 Now insertion sort.

Slide each card left, counting every card you look at, including the one that stops you. This is the Grade 3 method.

card put in	3	9	1	5	8	2	6	total
cards looked at								

3 Best and worst.

Starting hand of 8	insertion sort comparisons
already sorted	
exactly backwards	
our hand	

One of those ties selection sort exactly. Which, and why is that not a coincidence?

✎ ADD TO YOUR METHOD BOOK

Method Book page 2: both methods with their costs on our hand, and the note that selection sort always costs the same while insertion sort depends on the cards.

💬 TALK ABOUT IT

Selection sort never gets a lucky hand. Is that good or bad?

If you had to sort a pile that was **almost** in order already, which would you use?

STUCK? TRY THIS

For selection sort you do not need to sort anything to get the count. Just add 7 + 6 + 5 + ... + 1, because that is what the method does whatever the cards say.

★ CHALLENGE ZONE

Work out the selection sort cost for 20 cards and for 100 cards. Adding the run is slow, so look for the pairing trick: 1 + 20, 2 + 19, and so on.

Write a rule for the selection sort cost of **n** cards. You met this number in Grade 3, under a different name.

WEEK 3 STEPS & COST

Sorting by Splitting

Cut the pile in half, sort both halves, then zip them together.

IN CLASS THIS WEEK
Problem solving with whole numbers (5.OA.1, 5.NBT.5)

NUMBER WARM-UP · 2 MINUTES
Two sorted piles of 4. What is the most comparisons to merge them?

WORDS TO KEEP

merge zip two sorted piles into one by repeatedly taking the smaller top card

merge sort split, sort each half the same way, then merge

Merging two sorted piles is cheap. Each comparison places one card, so merging piles of **a** and **b** costs at most a + b − 1.

pile one	1	3	7	9				
pile two	2	5	6	8				
merged	1	2	3	5	6	7	8	9

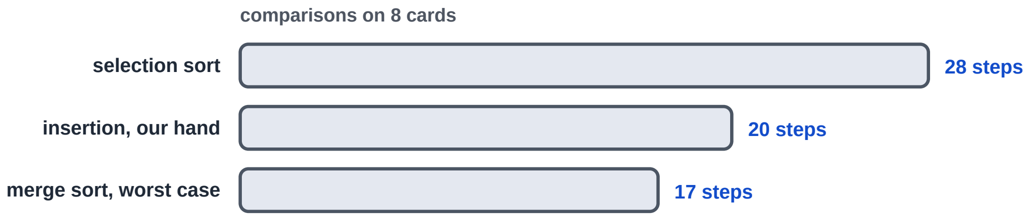
1 Count the merging.

a. Merging two piles of 1 card: at most ________ comparison. There are 4 such merges, so ________ in total.

b. Merging two piles of 2: at most ________. There are 2 such merges: ________

c. Merging two piles of 4: at most ________. Add all three rows: ________

2 Race it against the other two.



3 Now do it for a bigger pile.

Cards	selection sort	merge sort at worst
8	28	17
16		
64		

The gap grows. By 64 cards it is not close.

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 3: the three methods with their costs at 8 and at 64 cards. The 64 row is the one worth staring at.

TALK ABOUT IT

Merge sort does more **moving** than selection sort but far fewer comparisons. Is that a fair trade?

Why does splitting help at all? Nothing got easier, there is just more of it.

STUCK? TRY THIS

Work out one layer at a time. Count how many merges happen at that layer and how big each one is, then add the layers.

★ **CHALLENGE ZONE**

A pile of 1,024 cards. Selection sort costs about half a million comparisons. Work out roughly what merge sort costs.

How many layers of splitting does 1,024 cards take before every pile is a single card? You met that number in Grade 3.

WEEK 4 STEPS & COST

Searching a Sorted List

Halving is so much better than looking that the numbers stop feeling real.

IN CLASS THIS WEEK
Place value and powers of ten (5.NBT.1, 5.NBT.2)

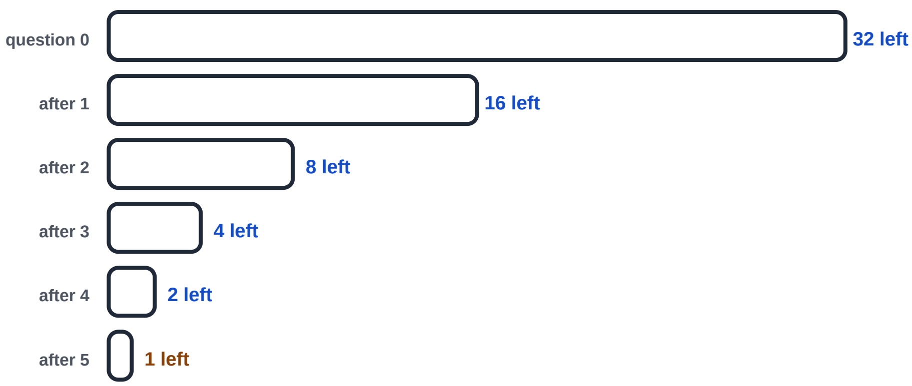
NUMBER WARM-UP · 2 MINUTES
Half of 1000, then half again, then again. How many halvings to reach 1?

WORDS TO KEEP

binary search look in the middle, throw away the half that cannot contain it, repeat

worst case the most steps a method could ever need, not the luckiest

A sorted list of **1,000** names. Each look at the middle throws away half of what is left.



1 Read the picture, then scale it up.

List length	looks needed at worst
32	5
1,000	
1,000,000	

The list gets a thousand times longer from 1,000 to 1,000,000. How much longer does the search get? ______________________

2 Against the slow way.

3 What halving needs.

a. Binary search only works if the list is ____________________

b. Sorting 1,000 names first costs about 10,000 comparisons. So is it worth sorting? It depends on how many times you search. Roughly how many searches before sorting pays for itself? _______

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 4: the halving table, and the sentence **a thousand times more names costs only ten more looks.**

💬 **TALK ABOUT IT**

The phone book is sorted and nobody complains. What does sorting cost the publisher, and who gets the benefit?

Why do we count the **worst** case rather than the average?

STUCK? TRY THIS

Do not divide. Just keep halving and count how many halvings it takes to get down to one, rounding up whenever you get a fraction.

★ **CHALLENGE ZONE**

A library has a billion books, sorted. How many looks to find one?

Somebody says halving is a thousand times faster than looking one at a time. On a list of 1,000, what is the real factor?

Doubling and Halving

The numbers behind last week. They get very big very fast.

IN CLASS THIS WEEK
Powers of ten and place value (5.NBT.2)

NUMBER WARM-UP · 2 MINUTES
2, 4, 8, 16. Keep going out loud as far as you can.

WORDS TO KEEP

power a number multiplied by itself over and over. 2 to the 5th means 2 × 2 × 2 × 2 × 2

doubling the opposite of halving, which is why last week and this week are the same week

Fill the doubling table. Every row is twice the row above.

2 to the	1	2	3	4	5	6	7	8	9	10
is	2	4								

1 Read it both ways.

a. 2 to the 10th is ________, which is just over ________________________

b. So halving a list of 1,024 down to 1 takes ________ halvings. Same number, read backwards.

2 Keep doubling.

2 to the	value
16	
20	
30	

Compare 2 to the 20th with one million. ______________________

3 Powers of ten, for comparison.

a. 10 to the 6th is ______________________

b. Roughly how many doublings does it take to pass one million? ________ That is the answer to last week's million-name search.

✎ ADD TO YOUR METHOD BOOK

Method Book page 5: the doubling table to 2 to the 20th, with 1,024 and 1,048,576 circled. You will reach for this page all year.

TALK ABOUT IT

Doubling twenty times passes a million. Does that surprise you? It surprises most adults.

Why does the doubling table answer a question about halving?

STUCK? TRY THIS

Do not multiply by 2 from the start each time. Take the row you just wrote and double it. One addition per row.

★ CHALLENGE ZONE

A penny on day 0, doubling every day. On which day does the pile first pass one million dollars? Careful with cents.

How many days until it passes the value of everything on Earth? You will need to guess what that is, and the answer is smaller than you expect.

WEEK 6 STEPS & COST

The Greedy Rule

Take the biggest that fits, then do it again. Simple, fast, and usually right.

IN CLASS THIS WEEK
Decimal operations with money (5.NBT.7)

NUMBER WARM-UP · 2 MINUTES
How would you make 68 cents with the fewest coins?

WORDS TO KEEP

greedy at every step take the biggest thing that still fits, and never look back

optimal the best possible answer, not just a good one

American coins: 25, 10, 5 and 1 cent. Make **68** cents with as few coins as you can.

25¢ 10¢ 5¢ 1¢

1 Run the greedy rule.

Work down from the biggest coin. For each one, take as many as fit before moving on.

Coin	how many	left to make
start		68
25 cents		
10 cents		
5 cents		
1 cent		0

Coins used altogether: ________

2 Three more.

Amount	greedy coins	how many
41 cents		
99 cents		
87 cents		

3 Is greedy the best here?

Take 99 cents. Try to beat the greedy answer by hand. Can you? ________ For American coins greedy always wins, which is lucky and not obvious. Next week is about a place where it does not.

✎ ADD TO YOUR METHOD BOOK

Method Book page 6: the greedy rule written as three lines, and the honest note that it is **not** proved to be best. It just happens to be, for these coins.

💬 TALK ABOUT IT

Greedy never looks back. What would a method that looked back have to do instead?

Why is greedy so easy to explain to somebody else?

STUCK? TRY THIS

Write down what is left after each coin. When what is left drops below the coin you are using, move down to the next coin.

★ CHALLENGE ZONE

Make 63 cents, then 64. One of them needs more coins than the other even though it is a smaller amount. Which, and why?

If the 5 cent coin did not exist, what would 68 cents cost in coins?

WEEK 7 STEPS & COST

When Greedy Fails

Change three coins and the simple rule starts giving wrong answers.

IN CLASS THIS WEEK
Decimal and whole number operations (5.NBT.7, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
With coins worth 1, 3 and 4, how would you make 6?

WORDS TO KEEP

counterexample one case where a method gives the wrong answer. One is enough

still useful a method that is usually right and sometimes wrong is not worthless, but you have to know which it is

A country with only three coins: **1**, **3** and **4**. Make **6**.

1 Greedy first.

a. Biggest that fits in 6: ________. Left to make: ________

b. Keep going. Greedy uses ________ coins in total.

c. Now find a better answer by hand. ______________________ How many coins? ________

2 Where else does it go wrong?

Same three coins. Fill in both columns and circle every row where greedy loses.

Amount	greedy coins	best possible
6		
8		
9		
11		
12		

3 So what is greedy worth?

Greedy was wrong on some of those and right on the rest. Write one sentence on when you would still use it.

✎ ADD TO YOUR METHOD BOOK

Method Book page 7, facing page 6: the coin set 1, 3, 4 and the amount 6. One counterexample is enough to demote a rule from **always** to **usually**.

TALK ABOUT IT

Last week greedy was perfect. This week it is not. What actually changed?

How would you find the true best answer, if greedy will not give it to you?

STUCK? TRY THIS

Do not trust the rule. Work out the greedy answer, then try to beat it by hand. If you can, greedy lost.

★ CHALLENGE ZONE

Invent your own three-coin set where greedy fails on some amount, and say which amount.

For the coins 1, 3 and 4, find every amount below 20 where greedy uses more coins than necessary. There are fewer than you would think.

WEEK 8 DOTS & LINES

Shortest Path With Costs

Not the fewest roads. The cheapest total, in dollars and cents.

IN CLASS THIS WEEK
Adding and subtracting decimals (5.NBT.7)

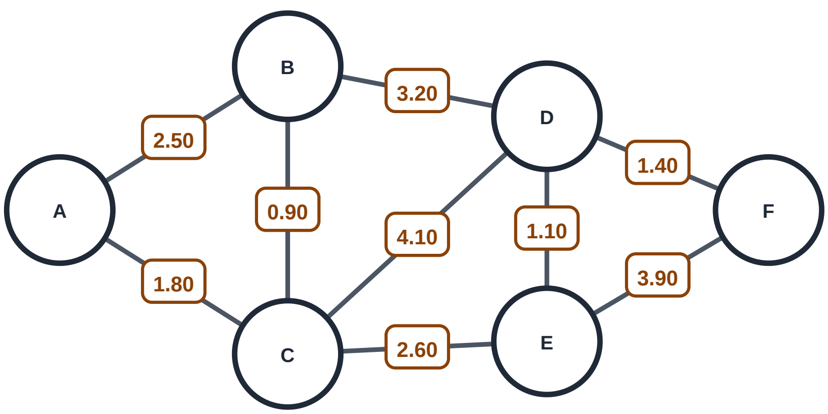
NUMBER WARM-UP · 2 MINUTES
What is 1.80 + 2.60? And 4.40 + 1.10?

WORDS TO KEEP

cost the number on a road, in dollars. The cheapest route is not always the one with the fewest roads

settled a town whose cheapest cost from A is known for certain, so it never changes again

Six towns. Every road shows its toll. Find the cheapest way from **A** to **F**.



1 Work outward from A, cheapest first.

Town	cheapest known cost from A	arrived from
A	0.00	
B		
C		
D		
E		
F		

2 Check the surprise.

a. The cheapest route to F costs ________

b. Write the towns it goes through. ______________________

c. How many roads is that? ________ Is there a route with **fewer** roads? ________ What does it cost? ________

3 Why settle in order.

Once a town has the cheapest cost of anything left, no later route can improve it, because every road costs something. Explain why that lets you finish each town and move on.

____

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 8: the map, your table, and the cheapest route marked. Note that it has more roads than the direct way and still costs less.

💬 **TALK ABOUT IT**

Fewest roads and cheapest cost gave different answers. Which would a delivery driver want?

What would go wrong with this method if a road could pay you to use it?

STUCK? TRY THIS

Always settle the unsettled town with the smallest cost so far. Then look at the roads out of it and see whether they make any neighbor cheaper than you already had.

★ **CHALLENGE ZONE**

Make the road from A to C cost 3.60 instead. Redo the table. Does the cheapest route to F change?

Add up the cost of every road on the map. Compare that with the cheapest route. What does the difference tell you about how much of a map a route actually uses?

The Cheapest Network

Two completely different methods. Same answer, every time.

IN CLASS THIS WEEK
Adding decimals; problem solving (5.NBT.7, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
Add 1.80 + 0.90 + 1.10 + 1.40. Do it in the easiest order you can find.

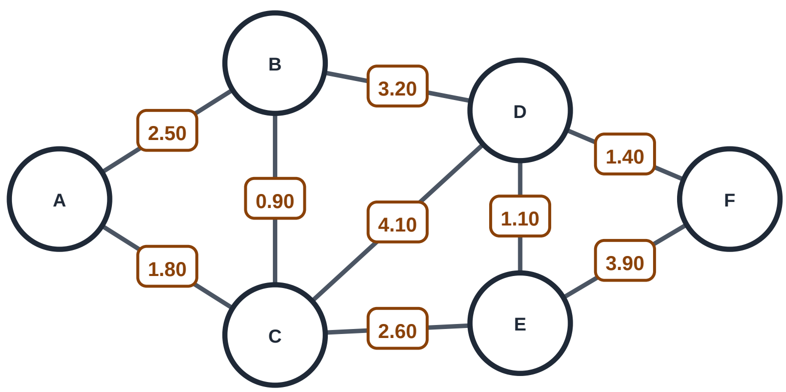
WORDS TO KEEP

network the roads you choose to build, so that every town is reachable

Kruskal take the cheapest road anywhere, unless it closes a loop

Prim start at one town and always take the cheapest road that reaches somewhere new

Same six towns. Now build roads so every town is reachable, for the least total money.



1 Method one: cheapest anywhere.

Sort every road by cost and take them in order, skipping any that would close a loop.

Roads taken, in order:

____

Total: ________

2 Method two: grow from A.

Start with only A. Each turn, take the cheapest road joining a town you have to a town you do not.

Roads taken, in order:

____

Total: ________

Same roads as method one? ________ Same total? ________

3 Count the work.

a. How many roads did each method end up with? ________ How many towns? ________

b. Kruskal has to sort all ________ roads first. Prim never sorts. Which is less work on a map with very many roads?

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 9: both methods side by side with their identical totals, and the note that the number of roads is always one fewer than the number of towns.

TALK ABOUT IT

Two methods that look nothing alike give the same answer. Does that mean they are secretly the same method?

If two roads cost exactly the same, could the two methods pick different ones?

STUCK? TRY THIS

Both methods refuse to close a loop. That is the only rule they really share, and it is the reason they land on the same total.

★ CHALLENGE ZONE

Change the road from B to D to cost 0.50 and rerun both methods. Do they still agree?

The cheapest route from A to F in Week 8 cost less than the cheapest network here. Explain why the network has to cost more even though it uses some of the same roads.

WEEK 10 STEPS & COST

Packing the Boxes

Same parcels, same boxes. Change the order they arrive in and you need a whole extra box.

IN CLASS THIS WEEK
Volume and measurement (5.MD.3, 5.MD.5)

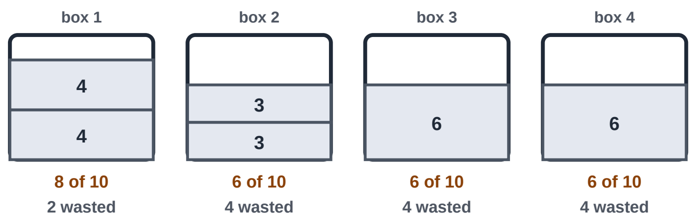
NUMBER WARM-UP · 2 MINUTES
4 + 4 + 3 + 3 + 6 + 6 = ?   How many boxes of 10 is that, at least?

WORDS TO KEEP

first fit put each parcel in the first box it fits in, and open a new box only when it fits nowhere

lower bound a number the answer cannot possibly go below. Here it is the total size divided by the box size, rounded up

Six parcels sized **4, 4, 3, 3, 6, 6**. Every box holds **10**. Parcels arrive in that order and you must place each one before the next appears.



1 What first fit did.

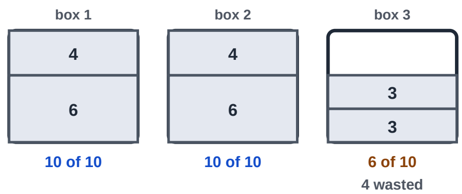
a. Boxes used: ________

b. Total size of all six parcels: ________

c. So the least number of boxes that could **possibly** work is ________. First fit used more.

2 Now sort them biggest first.

Same parcels, same rule, but deal with the big ones first: 6, 6, 4, 4, 3, 3.



Boxes used now: ________ Does that match the lower bound? ________

3 What changed.

The method did not change at all. Only the **order** changed. Write one sentence saying why big parcels first is the better order.

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 10: both packings drawn, the lower bound, and the sentence **sorting first cost nothing and saved a box.**

💬 TALK ABOUT IT

The lower bound proved 3 boxes was the target. Would it always be reachable?

A delivery firm cannot see tomorrow's parcels. Which method are they stuck with?

STUCK? TRY THIS

Work out the lower bound before you pack anything. If your packing matches it, you can stop, because nothing can do better.

★ CHALLENGE ZONE

Parcels sized 5, 5, 4, 4, 3, 3 into boxes of 10. What is the lower bound, and can you reach it?

Find six parcel sizes where sorting biggest first still does not reach the lower bound.

WEEK 11 STEPS & COST

Scheduling the Jobs

Six jobs, thirteen days of work, and it can all be done in ten.

IN CLASS THIS WEEK
Problem solving; measurement (5.OA.1, 5.MD.1)

NUMBER WARM-UP · 2 MINUTES
If digging takes 2 days and pouring takes 3 and pouring needs digging done first, how soon can pouring finish?

WORDS TO KEEP

depends on a job that cannot start until another one has finished

critical path the longest chain of jobs that depend on each other. It is exactly how long the whole thing takes

Building a shed. Some jobs wait for others, and some do not wait for anything.

Job	days	cannot start until
dig	2	nothing
pour	3	dig
frame	4	pour
paint	1	frame
plant	2	pour
sign	1	nothing

1 Find the longest chain.

a. Follow the depends-on column: dig, then pour, then frame, then paint. Add their days.

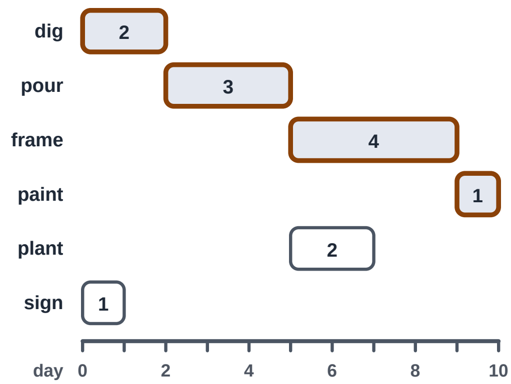
____

b. Add up **every** job's days. ________

c. Why is the finish date the first number and not the second?

____

2 Draw it on a calendar.



The outlined jobs are the critical path. The whole build finishes on day ______

3 Where to spend your money.

a. You can pay to make **one** job take a day less. Which job should you pick?

____

b. What happens if you speed up **plant** instead?

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 11: the schedule with the critical path marked, and the note that speeding up anything off that path buys you nothing at all.

💬 TALK ABOUT IT

Thirteen days of work finishing in ten. Where did the other three days go?

If you shorten the critical path enough, could a different chain become critical?

STUCK? TRY THIS

Work out the earliest each job can start by looking only at what it waits for. The last job to finish is your answer.

★ CHALLENGE ZONE

Make **frame** take 6 days instead of 4. What is the new finish date, and is the critical path still the same jobs?

Add a job **roof** taking 3 days that waits for frame. Redraw the schedule.

Method Fair

One problem, four methods, one bill each.

IN CLASS THIS WEEK
Problem solving (5.OA.1)

NUMBER WARM-UP · 2 MINUTES
No warm-up. Read all four rows first.

WORDS TO KEEP
fair test the same problem, the same size, and the same definition of a step for every method

The problem: put **64** numbered cards in order, then find one particular card. Fill in every cost, then decide.

Method	what it does	cost on 64 cards
selection sort	find the smallest, again and again	
merge sort	split, sort halves, merge	
look one at a time	check every card until you find it	
binary search	halve what is left, again and again	

1 Two of those sort and two of those search.

a. Which pair is a fair comparison? ______________________

b. Comparing a sort with a search would be unfair. Say why.

____

2 The real question.

a. You will search this pile **once**. Sort first, or just look? ________________________

b. You will search it **a hundred** times. Now what? ________________________

c. Roughly how many searches make sorting worth it? ________

3 Rank them.

✎ ADD TO YOUR METHOD BOOK

Method Book page 12: the four methods and their costs on 64 cards, and your answer to the sort-once-or-search-once question. That question is the whole trimester.

TALK ABOUT IT

A method that is slower but easier to explain: is it ever the right choice?

Why does the answer depend on how many times you are going to search?

STUCK? TRY THIS

Do not compare a sorting cost with a searching cost. Compare sorts with sorts and searches with searches, then ask what the whole job actually needs.

★ CHALLENGE ZONE

A shop looks up one price a day from a list of 1,000 items that changes every morning. Should they sort it?

Now the list changes once a year and they look up 500 prices a day. Same question.

WEEK 13 LOGIC & PROOF

Case Files

Everything from the first twelve weeks, priced.

IN CLASS THIS WEEK
Review and reasoning

NUMBER WARM-UP · 2 MINUTES
No warm-up this week.

Five cases. Each needs a number **and** the method that produced it.

1 The champion

Thirty cards. How many comparisons to find the biggest, and how do you know nothing can do better?

---

2 The two sorts

Selection sort on 12 cards costs how many? What is the most insertion sort could cost on the same 12?

---

3 The greedy coin

Coins 1, 3 and 4. Make 8. What does greedy give, and what is the true best?

---

4 The parcels

Parcels 7, 6, 5, 4, 4 into boxes of 12. What is the lower bound, and can you reach it?

___

5 The shed, again

If **pour** took 5 days instead of 3, when would the shed finish?

___

Which tool?

Beside each case write its method: lower bound ________, counting comparisons ________, greedy ________, first fit ________, critical path ________.

✎ **ADD TO YOUR METHOD BOOK**

Add a Method Book page called **My methods so far**, with the cost of each one written beside it in terms of n, wherever you can.

TALK ABOUT IT

Which of these five needed a lower bound to be sure?

Which method surprised you most by how cheap or expensive it was?

STUCK? TRY THIS

Before computing anything, ask what a step **is** for that case. Comparisons, boxes and days are three different currencies.

★ **CHALLENGE ZONE**

Write a sixth case whose answer is a lower bound rather than a method. Include an answer key on a separate sheet.

Counting on a Grid

Streets on a coordinate grid, and only two directions allowed.

IN CLASS THIS WEEK
The coordinate plane (5.G.1, 5.G.2)

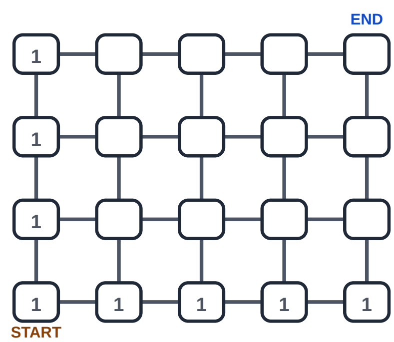
NUMBER WARM-UP · 2 MINUTES
From (0, 0) to (2, 1) going only right or up. How many routes?

WORDS TO KEEP

coordinates (across, up), counted from the corner at (0, 0)

route a walk from START to END going only right or up, never back

A grid **4 across** and **3 up**. START is at (0, 0) and END is at (4, 3). Write the number of routes into every corner.



1 Build it up.

a. Why is every corner along the bottom edge a 1?

____

b. Fill in every other corner by adding the one below and the one to its left. Routes to END: ________

2 Read some corners off.

Corner	routes to it
(2, 1)	
(3, 1)	
(2, 2)	
(4, 3)	

3 Every route is the same length.

a. How many steps right does any route take? ________ How many up? ________

b. So every route is exactly ________ steps long, and a route is just a decision about **which** of those steps go up.

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 13: the filled grid with its 35, and the note that a route is a choice of which 3 of the 7 steps go up. That is a Grade 4 counting question wearing a map.

💬 **TALK ABOUT IT**

Adding two corners is cheap. Drawing every route is not. How many would you have had to draw?

The numbers in the grid are Pascal's triangle tipped over. Where is row 7 in your grid?

STUCK? TRY THIS

Work outward from the bottom left, never from END backward. Every number you need is already written by the time you reach it.

★ **CHALLENGE ZONE**

How many routes across a grid 5 across and 3 up?

A grid 4 across and 4 up. Predict the answer from the choosing idea before you fill in the corners, then check.

WEEK 15 WAYS OF COUNTING

Paths That Avoid a Block

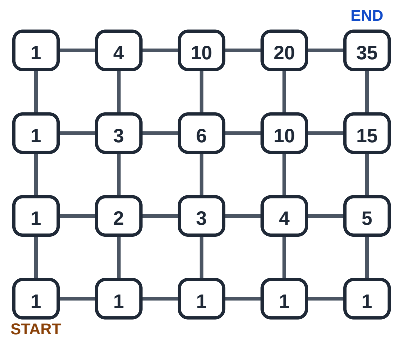
Count everything, then subtract the ones you did not want.

IN CLASS THIS WEEK
The coordinate plane; problem solving (5.G.1, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
If 35 routes cross a grid and 18 of them go through one corner, how many avoid it?

WORDS TO KEEP
the subtraction principle counting the bad ones and taking them away is often far easier than counting the good ones directly

The same 4 by 3 grid, but the corner at **(2, 1)** is flooded and no route may pass through it.



1 Count the bad routes.

a. Routes from START to (2, 1): ________ Read it off the grid.

b. Routes from (2, 1) on to END: ________ That is a smaller grid, 2 across and 2 up.

c. Every bad route is one of each, so bad routes = ________ × ________ = ________

2 Subtract.

a. Routes altogether: ________

b. Routes avoiding the flood: ________ − ________ = ________

3 Check it the slow way.

Fill in a fresh grid, but write **0** at (2, 1) and carry on adding as normal. You should land on the same answer.

✎ ADD TO YOUR METHOD BOOK

Method Book page 14: both ways of doing it, and the note that multiplying the two halves works because every bad route is one choice of each half.

TALK ABOUT IT

Why do you multiply the two halves rather than add them?

When would counting the good routes directly be easier than subtracting?

STUCK? TRY THIS

A route through the flooded corner is a route **to** it followed by a route **from** it. Count the two halves separately and multiply.

★ CHALLENGE ZONE

Flood the corner at (1, 1) instead. How many routes survive?

Flood two corners, (1, 1) and (3, 2). Be careful: this one is not just two subtractions.

WEEK 16 WAYS OF COUNTING

Two Circles, Then Three

Add the three clubs and you have counted some children twice and some three times.

IN CLASS THIS WEEK
Problem solving with whole numbers (5.OA.1)

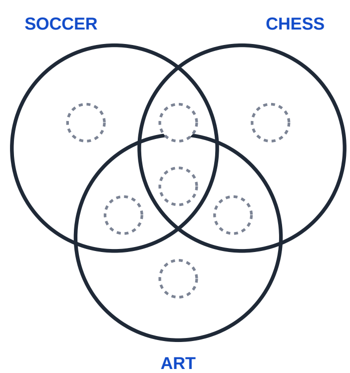
NUMBER WARM-UP · 2 MINUTES
18 + 12 + 14 = ?   Is that more than the 30 children in the class?

WORDS TO KEEP

overlap children in more than one club

inclusion and exclusion add the parts, subtract the double counted, add back the triple counted

A class of **30**. Soccer has 18, chess has 12, art has 14. 7 do soccer and chess, 8 do soccer and art, 5 do chess and art, and 3 do all three.



1 Fill the seven regions.

Start in the **middle** and work outward. The middle is 3. The soccer-and-chess petal is 7 − 3 = ________, because the 7 already includes the middle.

2 Add it up.

a. Add your seven regions. ________

b. Now the short way: 18 + 12 + 14 − 7 − 8 − 5 + 3 = ________

c. Same answer? ________ So how many of the 30 children do **no** club? ________

3 Why add the middle back.

A child in all three was counted 3 times by the first three numbers and taken away 3 times by the next three. So they are in the total ________ times, which is why you add them back once.

✎ ADD TO YOUR METHOD BOOK

Method Book page 15: the filled diagram and the formula, with the middle-child argument written out. Grade 2 did two circles. Three is where the pattern becomes a rule.

💬 TALK ABOUT IT

With only two clubs the formula is shorter. Write it.

Guess the formula for four clubs. You do not have to be right, but say why you guessed it.

STUCK? TRY THIS

Fill the middle first, then the petals, then the outer parts. Every number given to you already includes the regions inside it.

★ CHALLENGE ZONE

In a class of 40: 22 play soccer, 18 play chess, 9 play both. How many play neither?

Add art with 15 players, 6 in soccer and art, 5 in chess and art, 3 in all three. Now how many play nothing?

Counting the Leftovers

Sometimes the fastest way to count what you want is to count what you do not.

IN CLASS THIS WEEK
Place value and multi-digit numbers (5.NBT.1, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
How many three-digit numbers are there altogether?

WORDS TO KEEP

the complement everything that is not what you asked for

at least one the words that almost always mean: count the ones with none, and subtract

How many three-digit numbers contain **at least one 7**? Counting them directly is horrible. Counting the ones with no 7 at all is easy.

1 Count everything first.

a. Three-digit numbers run from 100 to 999. How many is that? ________

b. Now the ones with **no** 7. The first digit can be ________ things (not 0, not 7). The second can be ________. The third can be ________.

2 Multiply, then subtract.

a. Numbers with no 7: ________ × ________ × ________ = ________

b. So numbers with at least one 7: ________ − ________ = ________

3 Two more.

Question	all of them	the ones with none	answer
three-digit numbers with at least one 5			
four-digit numbers with at least one 0			

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 16: the phrase **at least one** and the instruction it should trigger: count them all, count the ones with none, subtract.

TALK ABOUT IT

Why is counting the numbers with at least one 7 directly so much harder?

Would this trick help for **exactly** one 7? Try it and see where it breaks.

STUCK? TRY THIS

The first digit of a three-digit number cannot be 0, so it never has the same number of choices as the other two. Getting that wrong is the usual mistake.

★ **CHALLENGE ZONE**

How many three-digit numbers have at least two 7s? The subtraction trick does not go straight through, so count them directly.

How many four-digit numbers contain no digit bigger than 5?

WEEK 18 FRACTIONS

Sharing Fairly

Dividing by a fraction is a counting question in disguise.

IN CLASS THIS WEEK
Dividing with unit fractions (5.NF.7)

NUMBER WARM-UP · 2 MINUTES
How many quarters are there in 1? In 2? In 3?

WORDS TO KEEP
how many fit 3 ÷ one fourth asks how many fourths fit inside 3
share among one fourth ÷ 3 asks something completely different

Three whole ribbons, cut into pieces one **fourth** long. How many pieces do you get?

1 Count the pieces.

a. Pieces from one ribbon: ________

b. From three ribbons: ________

c. Write it as a division: 3 ÷ one fourth = ________

2 The other question.

a. Now one fourth of a ribbon is shared between 3 people. How much does each get?

____

b. One of your answers is bigger than 3 and the other is smaller than one fourth. Say why dividing does both.

____

3 Four to try.

Division	in words	answer
4 ÷ one half	how many halves in 4	
2 ÷ one sixth		
one third ÷ 2		
one half ÷ 4		

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 17: the two questions side by side, with a drawing for each. Nearly everybody who gets this wrong has answered the other one.

💬 **TALK ABOUT IT**

Dividing usually makes things smaller. Here it made 3 into 12. What happened?

Read 6 ÷ one third out loud in words. Which of the two questions is it?

STUCK? TRY THIS

Say the division out loud as a question. **How many fit inside** is one question and **shared between** is the other, and they are not the same.

★ **CHALLENGE ZONE**

A 5 meter rope is cut into pieces two fifths of a meter long. How many pieces?

Write a story problem for 8 ÷ one quarter, and another for one quarter ÷ 8.

WEEK 19 FRACTIONS

A Fraction of a Fraction

Cut a square one way, then the other. The overlap is the answer.

IN CLASS THIS WEEK
Multiplying fractions as area (5.NF.4, 5.NF.6)

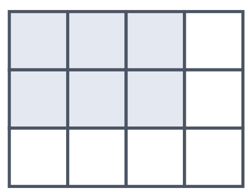
NUMBER WARM-UP · 2 MINUTES
What is one half of one half? Draw it before you answer.

WORDS TO KEEP

of in fractions, **of** means multiply

area model cut a square by one fraction across and the other down. The overlap is the product

One whole square. Cut it into **fourths** going across and take 3. Then cut it into **thirds** going down and take 2. The shaded overlap is 2 thirds of 3 fourths.



2 over 3 of 3 over 4

shaded: 6

squares in all: 12

1 Read the picture.

a. How many small squares are shaded? ________

b. How many small squares in the whole? ________

c. So 2 thirds of 3 fourths = ______________________, which simplifies to ______________________

2 Where the rule comes from.

a. The shaded rectangle is ________ across by ________ down, so its area is the tops multiplied.

b. The whole grid is ________ by ________, so its area is the bottoms multiplied. That is the whole rule.

3 Three more.

Question	shaded	in all	answer
one half of 2 fifths			
3 fourths of 2 thirds			
2 fifths of 3 fourths			

✎ ADD TO YOUR METHOD BOOK

Method Book page 18: the shaded square with both counts written on it, and the rule **tops times tops over bottoms times bottoms**, derived rather than announced.

TALK ABOUT IT

Multiplying usually makes things bigger. Both your answers were smaller than what you started with. Why?

Rows 2 and 3 of the table gave the same answer. Is that a coincidence?

STUCK? TRY THIS

Do not multiply first. Draw the grid, shade the overlap, and count. The rule should fall out of the picture rather than the other way round.

★ CHALLENGE ZONE

Work out 4 fifths of 5 eighths and simplify it. The answer is prettier than you expect.

Draw a square showing one half of one half of one half. What fraction is it?

WEEK 20 NUMBER STRUCTURE

Powers and Place Value

Every column is ten times the one on its right, and that is a rule you can push around.

IN CLASS THIS WEEK
Powers of ten and place value (5.NBT.2)

NUMBER WARM-UP · 2 MINUTES
What is 10 × 10 × 10? Write it as a power of ten.

WORDS TO KEEP

power of ten 10 to the 3rd means 10 × 10 × 10 = 1,000. The little number counts the zeros

shifting multiplying by 10 to the 3rd moves every digit three columns to the left

The columns, written as powers of ten.

1000	100	10	1
3	4	0	7

each column is 10 times the one to its right

1 Multiply by shifting.

Start	× or ÷	answer
2.5	× 10 to the 3rd	
340	÷ 10 to the 2nd	
0.06	× 10 to the 4th	
9,100	÷ 10 to the 3rd	

2 Say what the shifting is.

a. Multiplying by 10 to the 3rd moved every digit ________ columns to the ________________________

b. Dividing by 10 to the 2nd moved them ________ columns to the ________________________

3 Why the trick works.

A digit in the hundreds column is worth 100 times a digit in the ones column. Explain, using that, why multiplying by 100 is the same as sliding everything two places.

____

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 19: the power-of-ten columns, and the sentence **multiplying by a power of ten does not change any digit, only which column it sits in.**

💬 **TALK ABOUT IT**

Why does the decimal point look like it moves when really the digits do?

What would 10 to the 0 have to be, if every column is ten times the one to its right?

STUCK? TRY THIS

Count the zeros. 10 to the 4th has four zeros, so every digit shifts four columns. Do not count the places you move the point, count the columns the digits move.

★ **CHALLENGE ZONE**

Which is bigger, 3.6 × 10 to the 4th or 360 × 10 to the 2nd?

Write 47,000 as a number between 1 and 10 multiplied by a power of ten.

WEEK 21 NUMBER STRUCTURE

Base Five

A whole number system with only five digits. Not a code, a way of counting.

IN CLASS THIS WEEK
Place value and powers (5.NBT.1, 5.NBT.2)

NUMBER WARM-UP · 2 MINUTES
In base five the only digits are 0, 1, 2, 3 and 4. What comes after 4?

WORDS TO KEEP

base five every column is **five** times the one on its right, instead of ten

digit in base five a digit is never 5 or more, the same way base ten never uses a single digit for 10

The base five columns. Same idea as base ten, different multiplier.

125	25	5	1
2	0	3	1

each column is 5 times the one to its right

1 Read the number.

a. That is 2 lots of ________, 0 lots of ________, 3 lots of ________ and 1 lot of ________

b. In base ten that is ________

2 Both directions.

Base five	base ten
2031	266
400	
1234	
	63
	100

For the last two, work out how many 125s fit, then how many 25s, then 5s, then 1s.

3 Counting out loud.

Count from 1 to 12 in base five: 1, 2, 3, 4, ________, ________, ________, ________, ________, ________, ________, ________

✎ ADD TO YOUR METHOD BOOK

Method Book page 20: the base five columns beside the base ten ones, and the sentence **the digits mean nothing without knowing the base.**

💬 TALK ABOUT IT

Grade 2 wrote numbers in base two. What are the columns worth there?

Why is there no single digit for five in base five?

STUCK? TRY THIS

To go from base ten, ask how many 125s fit, write that digit, and carry on with what is left. It is the greedy rule from Week 6, on place values instead of coins.

★ CHALLENGE ZONE

Write 200 in base five, and 500.

In which base does the number 100 mean twenty-five?

WEEK 22 NUMBER STRUCTURE

Carrying in Base Five

Column addition works in any base. Only the moment you carry changes.

IN CLASS THIS WEEK
Adding within a place value system (5.NBT.7, 5.NBT.1)

NUMBER WARM-UP · 2 MINUTES
In base five, what is 3 + 4? It is more than 4, so it needs two columns.

WORDS TO KEEP
carry when a column reaches the base, write the leftover and carry 1 to the next column. In base ten you carry at 10, in base five at 5

Add **23** and **14**, both written in base five.

125	25	5	1
None	None	2	3

each column is 5 times the one to its right

1 Ones column first.

a. 3 + 4 = ________ in base ten. Is that 5 or more? ________

b. So take away 5, write ________, and carry ________

c. Fives column: 2 + 1 + the carry = ________. The answer is ________________________ in base five.

2 Check it in base ten.

a. 23 in base five is ________ in base ten. 14 in base five is ________

b. Add those. ________ Now convert your base five answer. ________ Match? ________

3 Three more.

Base five sum	answer in base five	check in base ten
12 + 13		
34 + 22		
444 + 1		

✎ ADD TO YOUR METHOD BOOK

Method Book page 21: the base five addition worked in full, with the base ten check beside it. The last row of the table is the one to look at twice.

💬 TALK ABOUT IT

What does 444 + 1 do in base five? Compare it with 999 + 1 in base ten.

Would subtraction with borrowing work the same way? Try 32 − 14 in base five.

STUCK? TRY THIS

Do each column in ordinary base ten arithmetic, then ask whether the answer reached five. If it did, take five away and carry one.

★ CHALLENGE ZONE

Add 1234 and 4321 in base five.

In base two, every column carries at 2. Add 1011 and 0110 in base two, and check your answer in base ten. Next week is about exactly this.

WEEK 23 STEPS & COST

How a Machine Adds

Two digits, one carry, and that is the entire arithmetic of a computer.

IN CLASS THIS WEEK
Place value in another base (5.NBT.1)

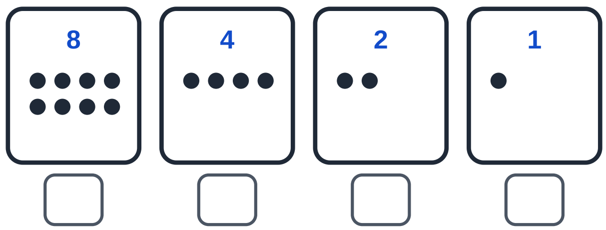
NUMBER WARM-UP · 2 MINUTES
In base two the only digits are 0 and 1. What is 1 + 1?

WORDS TO KEEP

bit one base two digit, 0 or 1

carry out the 1 that moves to the next column when a column reaches 2

The base two columns. Every one is **double** the one to its right.



1 One column at a time.

Adding two bits has only four cases. Fill in both answers for each.

bit	bit	write	carry
0	0		
0	1		
1	0		
1	1		

2 Now a whole sum.

a. Add 1011 and 0110 in base two, column by column, carrying whenever a column reaches 2.

____

b. Your answer has how many bits? ________ The two you started with had four each.

3 Check it.

a. 1011 in base ten is ________. 0110 is ________

b. Add them: ________ Convert your base two answer: ________ Match? ________

✎ ADD TO YOUR METHOD BOOK

Method Book page 22: the four-case table. A machine that can do those four rows can add any two numbers of any size, and it does nothing else.

💬 TALK ABOUT IT

Only four cases exist for one column. Why does that make base two good for machines?

The carry from one column becomes an input to the next. What does that mean for doing all the columns at the same time?

STUCK? TRY THIS

Do the rightmost column first and write the carry above the next column, exactly as you do in base ten. Nothing about the method changes, only when you carry.

★ CHALLENGE ZONE

Add 1111 and 0001 in base two. Watch how far the carry travels.

How many bits do you need to write the number 100? And 1,000?

Logic Gates

Three tiny machines. Wire them together and you get last week's adding.

IN CLASS THIS WEEK
Reasoning and patterns (5.OA.1, 5.OA.3)

NUMBER WARM-UP · 2 MINUTES
If it is raining AND I have an umbrella, I stay dry. When is that false?

WORDS TO KEEP

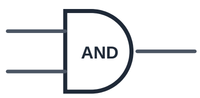
gate a tiny machine with inputs that are 0 or 1 and one output that is 0 or 1

AND output 1 only when both inputs are 1

OR output 1 when at least one input is 1

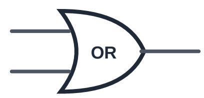
NOT output is the opposite of the input

Fill in the output column of each gate.



in	in	out
0	0	
0	1	
1	0	
1	1	

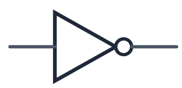
fill in the out column



in	in	out
0	0	
0	1	
1	0	
1	1	

fill in the out column

NOT



in	out
0	
1	

fill in the out column

1 Read them back.

a. AND outputs 1 in ________ of its four rows.

b. OR outputs 1 in ________ of its four rows.

2 Build the adder.

Look back at last week's four-case table. Compare its two answer columns with the gates.

a. The **carry** column matches one gate exactly. Which? ______________________

b. The **write** column is almost OR, but differs on one row. Which row?

____

c. That gate is called **exclusive or**: one or the other, but not both.

3 Two gates in a row.

Send both inputs through AND, then send that through NOT. Write the four outputs.

____

This one has a name too: NAND.

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 23: the three truth tables, and the note that **carry** is AND and **write** is exclusive or. That is a whole adder, in two gates.

💬 **TALK ABOUT IT**

A gate has no memory and no arithmetic in it at all. How does adding come out of that?

Could you build OR out of AND and NOT? Try it on all four rows.

STUCK? TRY THIS

Work one row of the table at a time and forget the picture. A gate is only ever a rule about what comes out for each pair going in.

★ **CHALLENGE ZONE**

Build a gate that outputs 1 only when both inputs are 0. Which gates do you need?

You met exclusive or in Grade 2, in the secret codes week, without its name. Look back and find it.

WEEK 25 LOGIC & PROOF

Puzzle Fair

A little of everything from Weeks 14 to 24.

IN CLASS THIS WEEK
Review and reasoning

NUMBER WARM-UP · 2 MINUTES
No warm-up. Start wherever you like.

Six puzzles. Answer plus method, every time.

1 The blocked street

A grid 5 across and 2 up. The corner at (2, 1) is closed. How many routes survive?

---

2 The three clubs

Of 40 children, 22 swim, 18 run, 12 cycle, 10 swim and run, 6 swim and cycle, 5 run and cycle, 3 do all three. How many do none?

---

3 The missing digit

How many three-digit numbers contain no 9 at all?

---

4 The other base

What is 302 in base five, written in base ten? And what is 88 in base ten, written in base five?

5 The ribbon

A 5 meter ribbon is cut into pieces two thirds of a meter long. How many whole pieces, and what is left?

6 The two gates

Both inputs go through OR, and that goes through NOT. Write all four outputs.

✎ **ADD TO YOUR METHOD BOOK**

Add a Method Book page called **My methods, weeks 14 to 24**: building up a grid, subtracting the bad ones, inclusion and exclusion, the complement, other bases, and gates.

TALK ABOUT IT

Puzzle 3 is a complement question turned inside out. Which is it really asking?

Which puzzle would have been hardest without the method you used?

STUCK? TRY THIS

Two of these are counting-by-subtracting in disguise. Working out which two is most of the work.

★ **CHALLENGE ZONE**

Write a seventh puzzle that needs both the subtraction principle and inclusion and exclusion. Include an answer key on a separate sheet.

Flow Through a Network

Water, not routes. How much can you push from one end to the other?

IN CLASS THIS WEEK
Problem solving; volume (5.OA.1, 5.MD.5)

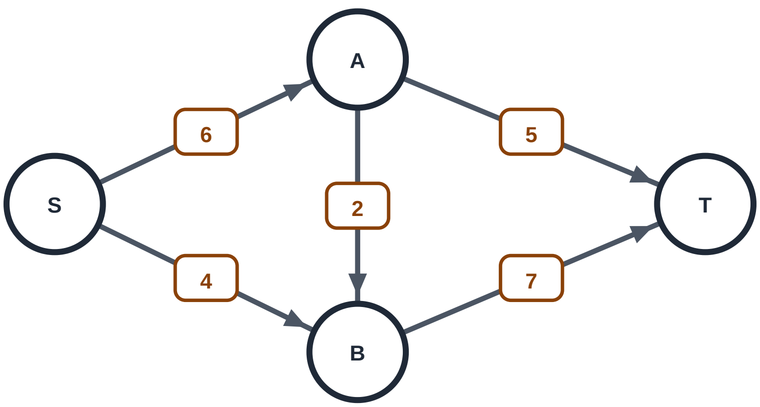
NUMBER WARM-UP · 2 MINUTES
A pipe carries 6 gallons a minute and another carries 4. Together, how much leaves?

WORDS TO KEEP

capacity the most a pipe can carry, written on the pipe

flow how much you actually send, which can never be more than the capacity

Water goes from **S** to **T**. Each pipe shows the most it can carry, in gallons a minute.



1 Send water down one path at a time.

Path	smallest pipe on it	send
S to A to T		
S to B to T		
S to A to B to T		

Total sent: ________ gallons a minute. Remember to subtract what you already used from each pipe before you take the next path.

2 Check against the ends.

a. Add the pipes **leaving S**: ________

b. Add the pipes **arriving at T**: ________

c. Your total cannot beat either of those. Which one did it match? ________________________

3 Why the third path was worth taking.

The pipe from A to B carries only 2 and looks useless. Say what it actually did for the total.

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 24: the network with your flow written on each pipe, and the total. Note which pipes ended up completely full.

💬 TALK ABOUT IT

Every pipe leaving S was full. Could any method have done better?

What would happen to the total if the pipe from A to B were removed?

STUCK? TRY THIS

Take one path at a time and send as much as the smallest pipe on it allows. Then reduce every pipe on that path by what you sent, and look for another path.

★ CHALLENGE ZONE

Change the pipe from S to B to carry 8. Does the total go up? Say why before you work it out.

Change A to T to carry 9 instead of 5. Now does the total go up?

The Bottleneck

The answer is never about the big pipes. It is about the narrow place.

IN CLASS THIS WEEK
Problem solving (5.OA.1)

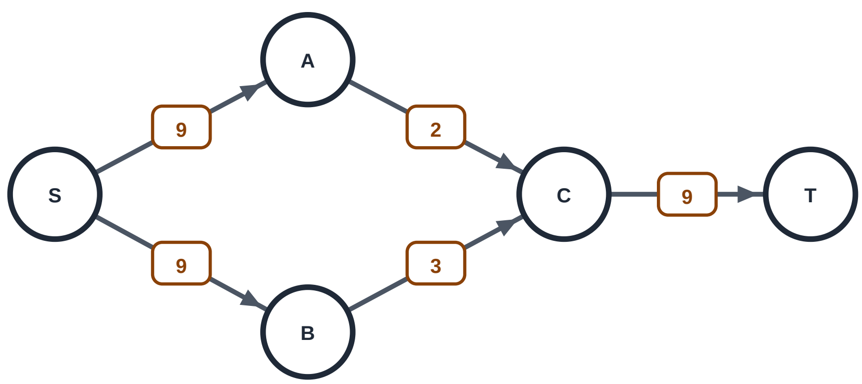
NUMBER WARM-UP · 2 MINUTES
Two wide pipes feed into one narrow one. What decides the total?

WORDS TO KEEP

a cut a line drawn across the network that separates S from T. Its size is the total capacity of every pipe crossing it

the bottleneck the smallest cut. No flow can ever beat it

Wide at both ends, narrow in the middle.



1 Try three cuts.

Cut S off from	pipes crossing it	total size
everything else	S to A, S to B	
A and B on the S side	A to C, B to C	
A, B and C on the S side	C to T	

Smallest cut: ________

2 Now push water through.

a. Send as much as you can from S to T. What total did you reach? ________

b. Compare it with the smallest cut. ________________________

3 Say the rule.

a. Every drop that reaches T has to cross **every** cut. So the flow can never be more than

____

b. It turns out you can always reach it exactly. Which pipes are the bottleneck here?

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 25: the network, the three cuts with their sizes, and the flow that matched the smallest one. This is a lower bound and an answer in the same page, which is the pattern from Week 1.

💬 TALK ABOUT IT

Making the pipes out of S wider would cost money and change nothing. Why?

Which single pipe would you widen, and by how much, to raise the total?

STUCK? TRY THIS

Do not look at the biggest pipes. Draw a line anywhere across the network and add up only the pipes that cross it. The smallest such line is your answer.

★ CHALLENGE ZONE

Widen A to C from 2 to 6. What is the new bottleneck and the new total?

Find a network where the smallest cut is neither at S nor at T.

Matching People to Jobs

Greedy gets stuck. Then you fix it without starting over.

IN CLASS THIS WEEK
Problem solving (5.OA.1)

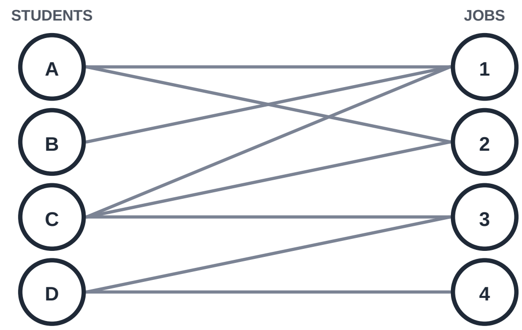
NUMBER WARM-UP · 2 MINUTES
If B can only do job 1 and A has already taken job 1, what should you do?

WORDS TO KEEP

matching giving out jobs so nobody shares one

swap chain when somebody is stuck, follow a chain of people who can shift over, and free up the job they need

Four students, four jobs. A line means that student can do that job.



1 Greedy, in alphabetical order.

Student	jobs they can do	greedy gives them
A	1, 2	
B	1	
C	1, 2, 3	
D	3, 4	

How many students got a job? ________ Who is left out? ________

2 Now fix it without starting again.

a. B needs job 1. Who has it? ________

b. Can that person move somewhere else? ________ Where? ________ Is it free? ________

c. Make the swap. Now how many students have jobs? ________

3 The finished matching.

A: ________ B: ________ C: ________ D: ________

✎ ADD TO YOUR METHOD BOOK

Method Book page 26: greedy's three, the swap chain drawn as arrows, and the four that came out of it. Greedy was not wrong, it was just not finished.

💬 TALK ABOUT IT

Greedy gave 3 and the true answer is 4. Was greedy wasted work?

Could a swap chain ever make things worse?

STUCK? TRY THIS

Start from the person with no job and follow the chain: who has the job they want, and where could that person go instead? Keep going until you reach a free job.

★ CHALLENGE ZONE

Take away C's line to job 3. Can all four still get jobs?

Draw four students and four jobs where greedy gives 2 and the true answer is 4.

WEEK 29 DOTS & LINES

Coloring, and How Good Is Greedy?

Six clubs, two slots are enough, and greedy uses three. The order is to blame.

IN CLASS THIS WEEK
Problem solving (5.OA.1)

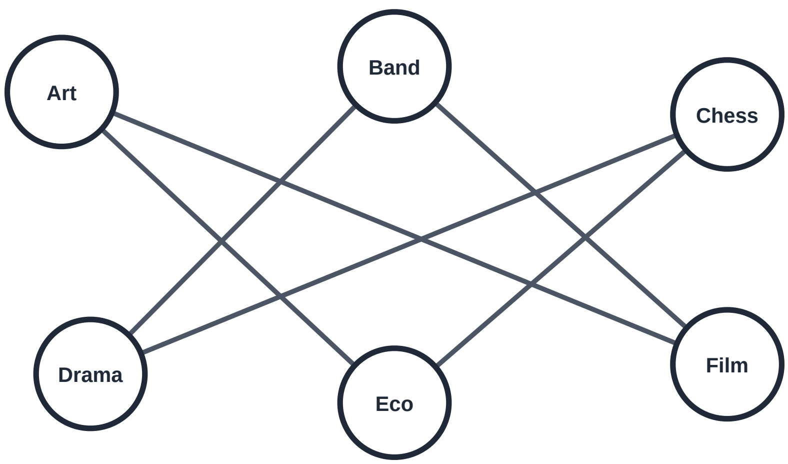
NUMBER WARM-UP · 2 MINUTES
Two slots are enough for a graph with no odd loop. Does greedy know that?

WORDS TO KEEP

slot a meeting time. Clubs joined by a line need different slots

greedy coloring go through the clubs in some order and give each one the lowest slot number that none of its neighbors already has

Six clubs. A line means they share a member. Notice there is no triangle anywhere in this picture.



1 Greedy, in a bad order.

Take the clubs in this order: **Art, Drama, Band, Eco, Chess, Film**. Give each the lowest slot its neighbors are not using.

Order	Art	Drama	Band	Eco	Chess	Film
slot	1					

Slots used: ________

2 Now a good order.

Same graph, same greedy rule, order **Art, Band, Chess, Drama, Eco, Film**.

Order	Art	Band	Chess	Drama	Eco	Film
slot	1					

Slots used: ________ Same method, same graph, different answer.

3 Which is right?

a. Two clubs joined by a line need different slots, so you can never do it in fewer than ________ slots.

b. Your second order reached that. So the true answer is ________, and the first order wasted a slot.

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 27: both orders and both answers, and the honest note that greedy coloring gives **an** answer, never necessarily **the** answer.

💬 **TALK ABOUT IT**

Greedy is fast and sometimes wrong. Dijkstra in Week 8 is also greedy and is never wrong. What is different?

Would sorting the clubs by how many neighbors they have help? Try it.

STUCK? TRY THIS

Work through the order one club at a time and write the slot down as you go. When you reach a club, look only at the neighbors that already have slots.

★ **CHALLENGE ZONE**

Find an order for these six clubs that makes greedy use **four** slots. It is harder than making it use three.

Draw a graph where greedy in the worst order uses five slots but two are enough.

WEEK 30 LOGIC & PROOF

A Lower Bound

Two halves make an answer: a method that works, and a proof nothing beats it.

IN CLASS THIS WEEK
Problem solving and reasoning (5.OA.1)

NUMBER WARM-UP · 2 MINUTES
If three rooms all clash with each other, how many slots do they need?

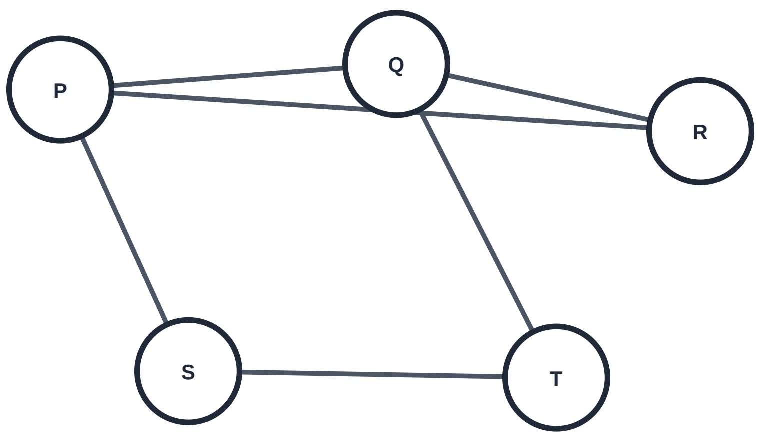
WORDS TO KEEP

upper bound a method that works. It proves the answer is no **worse** than this

lower bound an argument that no method can do better. It proves the answer is no **better** than this

settled when the two meet, the question is finished

Five rooms this time, and this graph **does** contain a triangle.



1 Find the lower bound first.

a. Find three rooms that all clash with each other. ________________________

b. Those three alone need ________ slots, so no method can use fewer.

2 Now the upper bound.

Room	P	Q	R	S	T
slot					

Slots used: ________ Does it match the lower bound? ________ Then you are done.

3 The same shape, three times this year.

Week	the method	the lower bound
1, finding the biggest	compare down the pile	
10, packing boxes	sort big first, then first fit	
27, the bottleneck	push water along paths	

✎ ADD TO YOUR METHOD BOOK

Method Book page 28: this week's two halves, and the table of the other three times the year did the same thing. Write at the top: **an answer is a method plus a reason nothing beats it.**

TALK ABOUT IT

You have found methods all year without lower bounds. What were those answers worth?

Which is usually harder to find, the method or the lower bound?

STUCK? TRY THIS

Find the lower bound before you look for a method. It tells you when to stop looking, which is the only thing that makes searching finite.

★ CHALLENGE ZONE

Add a sixth room that clashes with P, Q and R. What is the new lower bound?

Find a graph where the biggest all-clash group is 2 but three slots are genuinely needed. The lower bound is not always the whole story.

WEEK 31 WAYS OF COUNTING

Rules That Look Back

To count a strip of ten, count a strip of nine and a strip of eight.

IN CLASS THIS WEEK
Patterns and rules (5.OA.3)

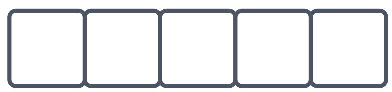
NUMBER WARM-UP · 2 MINUTES
A strip of 3 squares, tiled with 1s and 2s. List the ways.

WORDS TO KEEP

recurrence a rule that gives the next answer from the ones before it

base case the small answers you have to work out by hand before the rule can start

A strip of squares, covered by tiles that are **1** or **2** long. Count the different coverings.



a strip of five

1 Do the small ones by hand.

Strip length	1	2	3	4	5	6
coverings	1	2				

Look at your row. Each number is the ______________________

2 Why the rule is true.

a. Every covering of a strip of 6 ends in either a **1** tile or a **2** tile. There is no third option.

b. If it ends in a 1, the rest is a covering of a strip of ________

c. If it ends in a 2, the rest is a covering of a strip of ________

d. So the two cases add. That is the whole proof.

3 Push it further.

Carry the row on to a strip of **12**. You do not have to draw a single tile.

____

✎ **ADD TO YOUR METHOD BOOK**

Method Book page 29: the row of counts, the recurrence, and the ends-in-a-1-or-a-2 argument. Grade 2 counted these strips by drawing them. This is why you no longer have to.

TALK ABOUT IT

The rule needs two starting numbers rather than one. Why?

A strip of 20 has more than six thousand coverings. Could you have drawn them?

STUCK? TRY THIS

Do not draw the big strips. Work out lengths 1 and 2 by hand, then let every later answer come from the two before it.

★ **CHALLENGE ZONE**

Now allow tiles of length 1, 2 **and** 3. Work out the new rule and the first eight answers.

With tiles of 1 and 3 only, what is the rule? Be careful, it does not look back two.

A Rule From a Table

Two patterns started together. One is always double the other, and you can see why.

IN CLASS THIS WEEK
Generating patterns and graphing pairs (5.OA.3, 5.G.1)

NUMBER WARM-UP · 2 MINUTES
Start at 0 and add 3 five times. Now start at 0 and add 6 five times.

WORDS TO KEEP

term one number in a pattern

ordered pair (pattern A term, pattern B term), which can be plotted on a grid

Two patterns. **A** starts at 0 and adds 3. **B** starts at 0 and adds 6.

Step	0	1	2	3	4	5	6
pattern A	0	3					
pattern B	0	6					

1 Compare them.

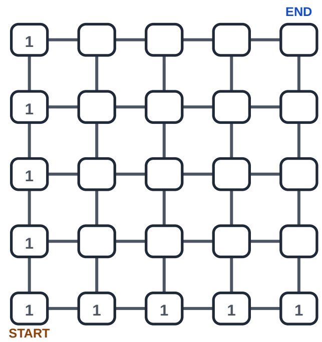
a. At every step, B is ____________________ A.

b. Say why, using the two adding rules rather than the numbers.

____

2 Plot the pairs.

Plot (A, B) for each step: (0, 0), (3, 6), and so on.



The points make a ____________________

3 A rule with a letter in it.

a. At step **n**, pattern A is ____________________

b. And pattern B is ____________________. Grade 6 spends a whole year on rules like these.

✎ ADD TO YOUR METHOD BOOK

Method Book page 30: both patterns, the doubling relationship, and the rule for step n. This is the first page in the book with a letter standing for a number.

💬 TALK ABOUT IT

Two patterns can be related without either one causing the other. Is that the case here?

If A added 3 and B added 5, would the points still lie on a straight line?

STUCK? TRY THIS

Compare the two patterns step by step, not term by term. The relationship comes from the adding rules, and it holds before you work out a single number.

★ CHALLENGE ZONE

Pattern C starts at 1 and adds 3. Is C always double anything?

Find two patterns where one is always three more than the other, and plot the pairs.

WEEK 33 LOGIC & PROOF

Two Rules, One Answer

A formula and a count that disagree cannot both be right, so check.

IN CLASS THIS WEEK
Reasoning about patterns (5.OA.3, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
$n \times (n - 1) \div 2$ for $n = 5$. Now count the handshakes for 5 people.

WORDS TO KEEP

verify check a rule against a case you counted by hand

one failure is enough a rule that works for six cases and fails on the seventh is not a rule

Three rules from this year. Test each one against a case you count by hand.

The rule	for n =	what the rule says	what you counted
finding the biggest costs $n - 1$	6		
selection sort costs $n(n - 1) \div 2$	5		
a tree with n dots has $n - 1$ lines	7		

1 Now a rule that is not one.

Somebody claims: **the number of coverings of a strip of n is always n**.

a. Test $n = 1$. Rule says ________, truth is ________

b. Test $n = 2$. ________ and ________

c. Test $n = 3$. ________ and ________

d. Test $n = 4$. ________ and ________. Where does it first fail? ________

2 Two cases are not enough.

That claim survived three tests and died on the fourth. Write one sentence about how many tests would be enough.

____

3 The one that is really true.

Check the strip recurrence for n = 7 against the row you built in Week 31. Rule: ________
Row: ________

✎ ADD TO YOUR METHOD BOOK

Method Book page 31: the three rules that passed and the one that failed, with the smallest case that killed it. A rule with no failing case tried against it has not been checked.

💬 TALK ABOUT IT

Grade 4 was about proving. This week is about checking. What is the difference?

How many cases would you want to see before trusting a new rule from somebody else?

STUCK? TRY THIS

Test the small cases first, especially n = 1 and n = 2. Rules that are wrong usually break at the very start or not until surprisingly late.

★ CHALLENGE ZONE

Somebody claims that a graph with n dots always has at most n lines. Find the smallest graph that breaks it.

Make up a rule that is true for n = 1 to 5 and false at n = 6.

WEEK 34 STEPS & COST

Sorting Really Big Lists

At eight cards it hardly matters. At a thousand it decides whether the job is possible.

IN CLASS THIS WEEK
Multi-digit operations; patterns (5.NBT.5, 5.OA.3)

NUMBER WARM-UP · 2 MINUTES
What is 1000 × 999 ÷ 2, roughly?

WORDS TO KEEP

growth how fast a cost rises as the list gets longer

the real question not which method is faster on your desk, but which one is still usable when the list is a thousand times bigger

The same two methods from Week 2 and Week 3, at four sizes.

Cards	selection sort	merge sort at worst
8	28	17
64		
1,024		

1 Look at how the two columns grow.

a. From 8 cards to 64 the list got ________ times longer. Selection sort got about ________ times more expensive.

b. Merge sort got about ________ times more expensive.

2 Draw the gap.

3 Put it in seconds.

A person can do about one comparison a second.

a. Selection sort on 1,024 cards would take about ________ seconds, which is about

____

b. Merge sort would take about ________ seconds, which is about ________________________

✎ ADD TO YOUR METHOD BOOK

Method Book page 32: the three-size table and the two times in human units. Six days against two and a half hours is the sentence to remember.

TALK ABOUT IT

At 8 cards merge sort saved 11 comparisons. Was it worth the extra complication then?

At what size would you switch? There is no single right answer.

STUCK? TRY THIS

Do not compute the exact numbers for 1,024. Work out roughly how many times bigger each column got, because the growth is the whole point.

★ CHALLENGE ZONE

A list of 1,000,000. Roughly how many comparisons does each method need?

A computer does a million comparisons a second. How long does each method take on the million-item list? One of these answers is measured in weeks.

WEEK 35 STEPS & COST

Which Method Would You Pay For?

The last real week. A decision, with money on it.

IN CLASS THIS WEEK
Decimal operations in context (5.NBT.7, 5.OA.1)

NUMBER WARM-UP · 2 MINUTES
What is 0.02 × 500? What is 130 × 0.02?

WORDS TO KEEP

one-off cost what you pay once to set something up

running cost what you pay every time you use it. The two are compared over how many times you expect to run it

A shop sorts a list of **10,000** prices every morning. Computer time costs **$0.20** for every million comparisons.

Method	comparisons	cost per morning
selection sort	49,995,000	
merge sort	123,617	

1 Work out the daily bill.

a. Selection sort: about 50 million comparisons, so ________ a morning.

b. Merge sort: about 0.12 million, so ________ a morning.

c. Difference over a 365 day year: ________

2 The other side of the ledger.

Writing merge sort costs a one-off **$400** of somebody's time. Selection sort is already written and costs nothing to set up.

a. How many days before merge sort has paid for itself? ________

b. So over one year, which is cheaper, and by how much? ________________________

3 Now change one number.

The shop only sorts **once a month** instead of daily. Over a whole year that is 12 sorts. Is the $400 worth spending now? Does your recommendation change?

____

✎ ADD TO YOUR METHOD BOOK

Method Book page 33, the last new page: the two costs, the payback point, and the fact that the right method changed when only the number of runs changed.

💬 TALK ABOUT IT

The faster method was not always the right choice. What made the difference?

What has this whole year been counting, if not seconds and not dollars?

STUCK? TRY THIS

Work out the cost of one run of each method first. Only then bring in the one-off cost, and ask how many runs it takes to cancel out.

★ CHALLENGE ZONE

The price of computer time falls to $0.02 per million, a tenth of what it was. Does the daily shop still write merge sort?

The list grows to 100,000 prices. Redo the daily costs. Notice which one grew more.

WEEK 36 STEPS & COST

The Method Book, Finished

Thirty-three pages of methods, each with its price. This week it becomes one book.

IN CLASS THIS WEEK
Review of the year

NUMBER WARM-UP · 2 MINUTES
No warm-up. Get every Method Book page out and put them in order.

Every page should have a **method** and a **cost**. A page with only a method is not finished.

1 The check list

Weeks	The page should say	✓
1 to 3	n − 1 and why nothing beats it; three sorts, priced	
4 to 5	halving; the doubling table	
6 to 7	greedy, and the coins where it fails	
8 to 9	cheapest route; two network methods, one answer	
10 to 12	first fit and the lower bound; the critical path	
14 to 17	grids; subtracting the bad ones; three circles; the complement	
18 to 22	dividing by a fraction; area models; base five	
23 to 24	how a machine adds; the three gates	
26 to 30	flow and the bottleneck; matching; greedy coloring; the two bounds	
31 to 35	looking back; a rule with a letter; growth; the money	

2 Four last decisions.

Name the method and say what it costs.

a. Find the largest of 500 numbers. ________

b. Find one name in a sorted list of 500. ________

c. Sort 500 numbers you will search once. ________________________

d. Sort 500 numbers you will search a thousand times. ________________________

3 Teach one page.

Pick the method you are proudest of and explain it out loud to somebody, including what it costs. Write down the first question they asked.

____

✎ ADD TO YOUR METHOD BOOK

Write a first page for the book: your name, the year, and one sentence saying what it means for one method to be better than another. Then number every page.

💬 TALK ABOUT IT

Which week changed how you decide things the most?

Grade 4 asked whether a thing is possible. This year asked what it costs. Which question do you find more useful?

STUCK? TRY THIS

A page you cannot price is not finished. That is the only test this week.

★ CHALLENGE ZONE

Look back at Week 1 and Week 30. Both found a method and then proved nothing could beat it. Write down what those two weeks have in common.

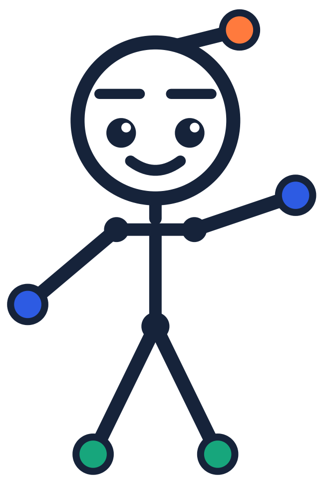
Invent a method of your own for something, work out its cost in steps, and put it on the last page of the book.

CERTIFICATE OF COMPLETION

Method Finder

awarded to

for a whole year of counting comparisons, racing three sorts against each other, halving the haystack, pricing the roads, packing the boxes, finding the bottleneck, and, hardest of all, proving that *nothing could have done it cheaper*.



a Young Innovator for Change

Teacher

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