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Structure Before Fluency

The case for discrete mathematics as enrichment in the early elementary grades

Young Innovators for Change · innovateyouth.org

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SUMMARY

Second grade mathematics is overwhelmingly about quantity: counting, place value, and fluency with addition and subtraction. Discrete mathematics — graphs, combinatorics, parity, algorithms — is about structure instead, and makes almost no demand on computational fluency. That property is pedagogically interesting: it decouples access to mathematical reasoning from arithmetic proficiency, at precisely the age when arithmetic proficiency begins to sort children into those who believe they are good at mathematics and those who do not.

This paper sets out the argument, reviews what is actually known, and is candid about what is not. The literature contains a sustained curricular case (Rosenstein, Franzblau & Roberts, 1997), credible existence proofs that seven- and eight-year-olds can do real graph theory (Hamkins, 2013; Blanco & García-Moya, 2021), and no efficacy trial at this grade level whatsoever. We describe the program we built against that evidence, the design constraints the evidence implies, and the study we propose to run — including the outcomes that would disconfirm our position.

Keywords: discrete mathematics, early elementary, mathematical practices, enrichment, equity of access, graph theory.

1 The problem this addresses

By the end of second grade a California student is expected to add and subtract fluently within 20, understand three-digit place value, and add and subtract within 1000. The Standards for Mathematical Practice ask for more than this — for argument, structure, and perseverance — but in most classrooms the practices are carried *on the back of* computational work. A child who is slow to regroup does not merely fall behind on regrouping. That child gets fewer opportunities to argue, to notice structure, and to be seen reasoning.

This is a sorting mechanism, and it operates early. The problem is not that arithmetic is over-emphasized; arithmetic matters and the fluency standards are correct. The problem is that arithmetic is nearly the *only* currency in which a seven-year-old can demonstrate mathematical competence, which means a narrow skill becomes a proxy for a broad identity.

Enrichment is the usual answer, and the usual enrichment makes the problem worse: it is offered to students who are already fast, and it is usually more arithmetic. What would help is content that is genuinely mathematical, genuinely hard, and *not* gated on computational speed.

2 Why discrete mathematics has a different load profile

Discrete mathematics studies structures that are separate and countable rather than continuous: graphs, sets, sequences, arrangements, algorithms. Three features make it unusually well suited to young children.

It is representationally concrete

A graph is dots and lines. A map coloring is crayons on regions. A parity argument is counters paired off on a desk. The objects of study are physically manipulable at an age when most mathematical objects are becoming abstract, which lets a child operate on the real thing rather than on a symbol standing for it.

The arithmetic demand is low and separable

Determining that a map needs three colors requires no computation at all. Determining that a figure cannot be traced in one stroke requires only the ability to count to five and distinguish odd from even. The mathematics is not made easy by this — the reasoning is genuinely demanding — but the *gate* is removed.

It produces impossibility, not just answers

This is the feature we consider most important and it is the least remarked upon. Ordinary elementary mathematics almost never asks a child to establish that something cannot be done. Discrete mathematics does so routinely and at an accessible level: two colors cannot suffice when three regions mutually border; five vertices cannot each have degree three, because the degree sum would be odd; the Königsberg walk cannot exist, because more than two landmasses have odd degree.

Each of these is a genuine proof, and each is within reach of a second grader who has been taught to count carefully. A child who has explained why something is impossible has done something categorically different from a child who has produced a correct answer, and they generally know it.

> **A NOTE ON WHAT IS NOT BEING CLAIMED**
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> We are not claiming that discrete mathematics improves arithmetic achievement. There is no evidence for that claim at this grade level and we have not designed a program that would produce it. The claim is narrower: that this content gives more children access to mathematical reasoning and to a mathematical identity, and that it does so without displacing curricular time from the standards.

3 What the evidence shows

3.1 The curricular argument is well established

The foundational statement is the DIMACS volume edited by Rosenstein, Franzblau and Roberts (1997), published jointly by the American Mathematical Society and the National Council of Teachers of Mathematics. Across thirty-four contributions it argues that discrete mathematics "can and should be taught in K–12 classrooms," and it includes material specifically addressed to the K–2 grade band. Its central move is worth noting: it positions discrete mathematics not as an additional topic competing for time, but as "a vehicle for achieving the broader goals" of mathematics education — a way of teaching reasoning, using content that happens to be interesting.

DeBellis and Rosenstein (2004) restate this for an international audience and describe discrete mathematics as "a vehicle for providing teachers with a new way to think about traditional mathematical topics." Their status report is, however, largely an account of publications and advocacy. It does not present substantial empirical data on elementary implementation, and the authors do not claim otherwise.

3.2 The existence proofs are credible and specific

Two accounts establish that the content is teachable at the target age.

Hamkins (2013) taught vertex coloring, chromatic numbers, map coloring, Eulerian paths and circuits, and the Seven Bridges of Königsberg to a **second-grade class of seven- and eight-year-olds**. Students colored graphs with a minimum number of colors, constructed challenge graphs for their classmates, designed their own maps, and traced paths without repeating edges. This is not an adjacent topic list; it is very nearly the content of Weeks 17, 18, 26 and 27 of the program described below, delivered to the same age group by a research mathematician, with students described as working independently and collaboratively and one announcing an intention to become a mathematician.

Blanco and García-Moya (2021) ran a workshop program covering map coloring and the four-colour theorem, Eulerian cycles and paths, complete graphs and the handshake problem, Königsberg, Hamiltonian paths, and vertex degree, with primary students aged **6–11**. They report that students deployed pattern recognition, trial and error, counting strategies, induction, use of symmetry, and guess-and-check, and that participants went on to invent analogous problems of their own — evidence of transfer rather than mere task completion. Their conclusion is that graph theory "successfully increases motivation of participants towards mathematics and allows the appearance and enforcement of problem-solving strategies."

LIMITATIONS WE ARE NOT GOING TO TALK AROUND

- **Blanco and García-Moya studied seven gifted students.** The sample is small and selected for high mathematical performance. It establishes feasibility and suggests motivational benefit; it says nothing reliable about a general-population classroom, which is exactly the population we care about.
- **Hamkins is an existence proof, not a study.** It is a well-documented account of a successful lesson by an unusually capable instructor. It demonstrates that the ceiling is higher than assumed. It does not demonstrate that a typical teacher gets a typical result.
- **The strongest methodological work is at the wrong grade.** Ferrarello, Gionfriddo, Grasso and Mammana (2022) is a careful study within the Teaching for Robust Understanding framework, showing that students can model real situations as graphs and reason about constrained arrangements — but its participants are **eighth graders**. It supports the accessibility of the content in school settings generally. It cannot be read down to second grade.
- **There is no efficacy trial.** We could locate no randomized or quasi-experimental study of a discrete mathematics enrichment program in the early elementary grades with measured outcomes. Anyone who tells you the evidence base here is strong is overstating it.

3.3 The adjacent literature is more developed

Computational thinking in the primary grades — algorithms, decomposition, pattern recognition, debugging — overlaps substantially with the algorithms and codes strand of discrete mathematics, and has attracted considerably more empirical attention, including structured-curriculum investigations in early-years primary schooling and a body of work on integrating computational thinking into PreK–5 settings. We regard this as supportive context rather than direct evidence: the overlap is real but partial, and outcome measures in that literature are largely computational-thinking constructs rather than mathematical reasoning or disposition.

4 Alignment with California policy

The revised *Mathematics Framework for California Public Schools*, adopted in November 2023, is unusually hospitable to this kind of content. Its organizing move is to group standards into "Big Ideas" that integrate rather than isolate concepts, and to foreground investigation. Its stated goal is to "ensure equity and excellence in math learning so that all California students become powerful users of mathematics," and it rests on the position that all students can become productive mathematics learners.

Two consequences follow for a program of this kind. First, an enrichment sequence built around investigation and argument is aligned with the framework's direction rather than tolerated alongside it. Second, and more practically, the Standards for Mathematical Practice — persevering in problem solving, explaining one's thinking, constructing arguments — are where discrete mathematics contributes most, and they are the part of the standards that classroom materials most often underserve.

Content-standard alignment is a secondary but real benefit. Because the pairing can be arranged deliberately, an enrichment week can be positioned to reinforce whatever the class is already teaching:

DISCRETE TOPIC	CA STANDARD REINFORCED	MECHANISM
Parity by pairing	**2.OA.3**	Odd and even developed from pairing; even numbers written as two equal addends
Loops in robot programs	**2.OA.4**	A loop of n repeats of k steps is k added n times
Binary cards	**2.NBT.1**	Place value with multiplier two rather than ten
Weighted shortest path	**2.NBT.5**	Repeated two-digit addition with a reason to compare the totals
Rectangle tilings	**2.G.2**	Rows and columns of same-size squares, counted both ways
Coin combinations	**2.MD.8**	Exhaustive enumeration over a money context

5 The equity argument, stated carefully

The equity claim we make is a claim about *access to the activity*, not about closing an achievement gap. It has three parts.

The prerequisite is lower. A student who has not yet secured regrouping is not excluded from reasoning about a map coloring. In a classroom where enrichment is normally allocated by arithmetic speed, this changes who is eligible.

The material cost is near zero. Crayons, counters, number cards, and a photocopier. No devices, no licences, no kits, no consumable refills. Programs that require purchase are adopted disproportionately by schools that can purchase.

It is legible to families. A parent who is not confident in mathematics can still ask a child why two colors will not work, and can be genuinely convinced by the answer. That is a different family interaction than checking a worksheet of sums.

> **THE COUNTER-ARGUMENT**
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> The serious objection is opportunity cost. Time spent on graph coloring is time not spent on fluency, and for a student already behind on fluency that trade may be the wrong one. We take this seriously, and it is the reason the program is built as a *weekly* 30–40 minute supplement rather than a daily replacement, and the reason every week opens with a short grade-level warm-up. It is also a question the pilot below is designed to answer rather than assume.

6 Design constraints the evidence implies

The literature is thin on outcomes but quite informative about failure modes. Five constraints followed directly, and each is a decision we made against a plausible alternative.

1. **The teacher must not need to learn graph theory first.** Hamkins' result was produced by a research mathematician. If a program depends on that, it does not scale. Every answer, every misconception, and every launch note therefore lives in a guide, not in the teacher's prior knowledge.
2. **Nothing may require preparation.** Every map, grid, graph, clock face, coin set and tiling is printed on the page. A program that requires assembling manipulatives will be run for three weeks and abandoned.

3. **It must survive a black-and-white copier.** Colour is accent only; structure lives in the linework. This is not an aesthetic preference. A program that needs a colour printer is a program for wealthy schools.
4. **The stuck moments must be protected.** The impossibility results are the distinctive contribution, and they only work if the child is allowed to fail first. The guide instructs teachers explicitly not to rescue early — a counter-intuitive instruction that must be given in as many words.
5. **It must connect to the pacing guide, or it will be cut.** Enrichment that reads as unrelated is the first thing dropped in April. Sequencing each week against the standard the class is already teaching is a survival mechanism as much as a pedagogical one.

7 What we built

Discrete Math Adventures is a 36-week Grade 2 enrichment program: one two-page assignment per week, 30–40 minutes, sequenced against a typical California Grade 2 pacing guide. Six strands — sorting and logic, number structure, counting, maps and colours, paths and routes, algorithms and codes — interleave so that each idea returns three or four times across the year with increasing weight.

Three weeks terminate in a proved impossibility. Parity, developed in Week 3, is the tool that resolves the Euler tracing problem in Week 26 and the Königsberg bridges in Week 27 — a deliberate twenty-three-week gap between acquiring an instrument and needing it. The complete materials are a student workbook, a teacher guide with full answer keys, and no other required component.

> **STATUS**
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> The program is designed and complete but **has not been evaluated**. It has not been run with a class. Nothing in this paper should be read as a claim about its effects.

8 The study we propose

The gap identified in §3 is one we intend to address rather than cite. A defensible first study is small, honest about its limits, and pre-committed to what would count as a negative result.

Design

A matched-classroom quasi-experiment across four to eight Grade 2 classrooms in the San Gabriel Valley, with classrooms rather than students as the unit of assignment, run over a single school year. Treatment classrooms receive the weekly assignment; comparison classrooms continue with their existing enrichment practice. A randomized design is not realistic at this scale; we will report it as what it is.

Measures

CONSTRUCT	INSTRUMENT	TIMING
Mathematical disposition	Established elementary attitude/identity scale, administered orally	Pre and post
Reasoning and justification	Task-based interview on two unfamiliar non-routine problems, scored by rubric	Pre and post
Grade-level achievement	The district's existing benchmark — no additional testing	District schedule
Participation breadth	Teacher log: which students volunteer explanations, by week	Continuous
Feasibility	Weeks completed; minutes spent; teacher exit interview	Continuous, then post

The participation-breadth measure is the one we care about most, because it operationalizes the actual hypothesis: that a different kind of content changes *who* gets to be the child explaining something to the class.

WHAT WOULD SHOW WE ARE WRONG

- Grade-level benchmark performance declines relative to comparison classrooms. This would mean the opportunity cost is real and the supplement is not affordable.
- Participation breadth does not move, or moves toward students already identified as high-achieving. This would mean we have built a better gifted program, not a wider door — the specific failure the Blanco and García-Moya sample warns of.
- Teachers do not complete the year. Below roughly two-thirds of weeks, the design constraints in §6 have failed regardless of what the other measures say.

Reporting

We will publish the protocol before data collection and report the results whether or not they favour the program. Given the sample size, the appropriate output is a descriptive feasibility and promise report, not an effect-size claim.

9 Conclusion

The argument for discrete mathematics in the early grades has been available for nearly thirty years, is endorsed in a joint AMS/NCTM volume, and has never been properly tested below middle school. Meanwhile the content demonstrably works in second-grade classrooms when someone competent teaches it, and it asks for exactly the reasoning that the current California framework says it wants.

The honest summary is that this is a well-motivated, under-evidenced idea with an unusually low cost of trying. That combination argues for a careful pilot, not for confident adoption — and not for waiting.

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A NOTE ON THE EVIDENCE IN THIS PAPER

Every source cited above was located and read for this paper. Where a study's population, grade level, or sample size limits what it can support, that limitation is stated in the text rather than in a footnote. Readers should treat §3.2's existence proofs as demonstrations of feasibility and the motivational findings as suggestive rather than established. No claim of measured effect is made for the program described in §7, because none has been measured.

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