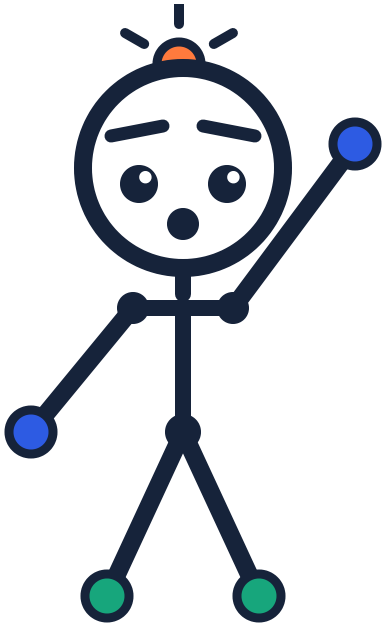
Young Innovators
FOR CHANGE
innovateyouth.org

A FULL-YEAR ENRICHMENT PROGRAM · GRADE 3

Count Every Way

36 weekly assignments on counting things properly, and knowing you did not *miss any*



This book belongs to

_____

Teacher _____ School year _____

One assignment a week · two pages, front and back · about 30 to 40 minutes
Built to sit alongside the California Grade 3 math curriculum, not to replace it.

How This Book Works

Read this page with a grown-up before Week 1.

Hi again. I am Pip.

Last year we looked at maps, robots and codes. This year has one question underneath all of it: **how many?** And a harder one right behind it, which is **how do you know you did not miss any?**

One assignment a week, two pages

Page 1 has a note about what you are doing in class this week, a two-minute **Number Warm-Up**, the **Words to keep**, and the main problem.

Page 2 has more to try, then **Add to your Counting Book**, a **Talk About It** question, a hint, and a ★ **Challenge Zone**.

The Counting Book

You will build one thing all year: a book of your own, one page at a time, in your own words. By Week 36 it is about thirty pages and it is the part you keep. Get a folder or a notebook for it in Week 1, not in Week 10.

Three rules for this book

1. An answer without a reason is only half done. Every week asks **how do you know**.
2. Being organized is not tidiness here, it is the actual method. A messy list is a wrong list.
3. Some questions have **no** answer, and proving that is harder and better than finding one.

For the grown-up helping

The single most useful thing you can say is **“show me you have them all.”** Most of this year is the difference between an answer that happens to be right and an answer the child can defend. When they are stuck, ask them to list the first three cases out loud in order. That unsticks nearly everything.

The Year at a Glance

Weeks 1–18. The last column is what the class is usually doing that week.

WK	ASSIGNMENT	STRAND	IN CLASS THIS WEEK
TRIMESTER 1 · WEEKS 1–12			
1	**In Order, Every Time**	Ways of Counting	Multiplication as equal groups (3.OA.1)
2	**Did I Miss Any?**	Logic & Proof	Multiplication and division (3.OA.3)
3	**Two Stages**	Ways of Counting	Equal groups (3.OA.1, 3.OA.3)
4	**Rows and Columns Count**	Ways of Counting	Arrays and area models (3.OA.3)
5	**Three Stages, One Tree**	Ways of Counting	Properties of multiplication (3.OA.5)
6	**Order Matters**	Ways of Counting	Multiplying three factors (3.OA.5)
7	**Order Does Not Matter**	Ways of Counting	The unknown factor (3.OA.6)
8	**Handshakes, Bigger**	Ways of Counting	Fluency within 100 (3.OA.7)
9	**Dots, Lines and Degree**	Dots & Lines	Fluency within 100 (3.OA.7)
10	**Pigeons and Holes**	Logic & Proof	Two-step word problems (3.OA.8)
11	**Sorting Your Hand**	Steps & Rules	Adding and subtracting within 1000 (3.NBT.2)
12	**Splitting the Search**	Steps & Rules	Rounding to the nearest 10 or 100 (3.NBT.1)
TRIMESTER 2 · WEEKS 13–24			
13	**Case Files**	Logic & Proof	Review & reasoning
14	**Cutting a Rectangle**	Shape & Space	Area by tiling (3.MD.6, 3.MD.7)
15	**Same Area, Different Shape**	Shape & Space	Area and perimeter (3.MD.7, 3.MD.8)
16	**Same Perimeter, Different Area**	Shape & Space	Perimeter (3.MD.8)
17	**Rectangles Inside a Grid**	Ways of Counting	Multiplication problems (3.OA.3)
18	**Factor Pairs**	Ways of Counting	Unknown factors (3.OA.6, 3.OA.4)

The Year at a Glance

Weeks 19–36. The last column is what the class is usually doing that week.

WK	ASSIGNMENT	STRAND	IN CLASS THIS WEEK
19	**Only Two Rectangles**	Ways of Counting	Multiplication fluency (3.OA.7)
20	**Sharing Without Remainder**	Ways of Counting	Division as equal groups (3.OA.2)
21	**What Is Left Over**	Number Structure	Interpreting remainders (3.OA.8)
22	**The Days Wrap Around**	Number Structure	Elapsed time (3.OA.8, 3.MD.1)
23	**Reading a Scaled Graph**	Data	Scaled bar graphs (3.MD.3)
24	**Counting From a Picture**	Data	Scaled picture graphs (3.MD.3)
TRIMESTER 3 · WEEKS 25–36			
25	**Puzzle Fair**	Logic & Proof	Review & reasoning
26	**Parts of a Whole**	Fractions	Fractions as equal parts (3.NF.1)
27	**Making One Whole**	Ways of Counting	Fractions on a strip (3.NF.1, 3.NF.3)
28	**Same Amount, Different Name**	Fractions	Equivalent fractions (3.NF.3)
29	**Coloring a Schedule**	Dots & Lines	Two-step problems (3.OA.8)
30	**Can Everyone Be Paired?**	Dots & Lines	Two-step problems (3.OA.8)
31	**Instructions With a Repeat**	Steps & Rules	Arithmetic patterns (3.OA.9)
32	**Instructions With a Question**	Steps & Rules	Two-step problems (3.OA.8)
33	**Counting Paths on a Grid**	Ways of Counting	Arithmetic patterns (3.OA.9)
34	**The Triangle of Numbers**	Ways of Counting	Arithmetic patterns (3.OA.9)
35	**Prove It With Pairing**	Logic & Proof	Even and odd; reasoning (3.OA.9)
36	**The Counting Book, Finished**	Ways of Counting	Review of the year

WEEK 1 WAYS OF COUNTING

In Order, Every Time

There is a way to list things so that nothing gets skipped.

IN CLASS THIS WEEK
Multiplication as equal groups (3.OA.1)

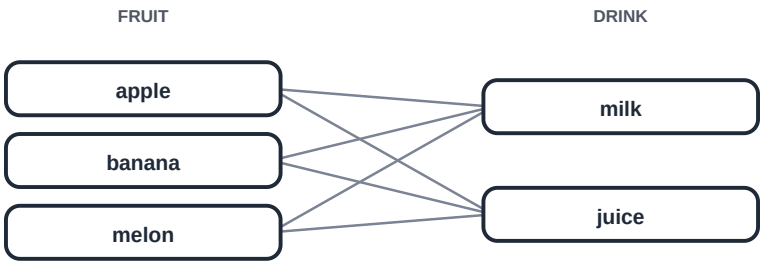
NUMBER WARM-UP · 2 MINUTES
Count by 3s out loud to 30. Then count by 4s to 40.

WORDS TO KEEP

systematic following one fixed order, on purpose, so nothing is skipped

list every single answer written down, one after another

Lunch is **one fruit** and **one drink**. Here is what is on the counter.



1 List every different lunch.

Do it **in this order**: all the **apple** lunches first, then all the **banana** ones, then all the **melon** ones. Inside each group, put **milk** before **juice**.

#	fruit	drink
1		
2		
3		
4		
5		
6		

How many lunches in all? _____

2 Now four fruits and two drinks.

Somebody added **pear** to the counter. You do not have to write the whole list again. Think about what one new fruit adds.

a. How many lunches now? _____

b. How do you know, without listing them all?

_____

3 Two digits, from 3, 5 and 7.

Make two-digit numbers using only the digits **3**, **5** and **7**. A digit may be used twice, so 55 is allowed. List them in order, smallest first.

_____

_____

How many did you get? _____

✎ ADD TO YOUR COUNTING BOOK

Write the sentence **To be sure I have them all, I list them in one fixed order** at the top of a fresh page. Under it, copy your apple, banana, melon list.

💬 TALK ABOUT IT

Why is it hard to know you are finished if you write the lunches down in whatever order they pop into your head?

If you had listed the **drinks** first instead of the fruits, would you end up with the same lunches? Would the list look the same?

STUCK? TRY THIS

Hold the first thing still. Change only the second thing until it runs out. **Then** move the first thing along by one and start the second thing over.

★ CHALLENGE ZONE

Make every three-letter arrangement of the letters **A**, **B** and **C**, using each letter exactly once. How many are there?

Now try it with **A**, **B**, **C** and **D**. This one is worth being organized about.

WEEK 2 LOGIC & PROOF

Did I Miss Any?

A list is not finished just because you stopped writing.

IN CLASS THIS WEEK
Multiplication and division within 100 (3.OA.3)

NUMBER WARM-UP · 2 MINUTES
What is 8 × 3? Now what is 3 × 8? Say why they match.

WORDS TO KEEP

complete nothing has been left out

duplicate the same answer written down twice

Mia was asked to list every way to pick **two different coins** from a penny, a nickel, a dime and a quarter. Here is her list.

#	Mia's pairs
1	penny and nickel
2	penny and dime
3	nickel and dime
4	nickel and quarter
5	penny and dime
6	dime and quarter

1 Two things are wrong with it.

a. One pair is written **twice**. Which one? _____

b. One pair is **missing**. Which one? _____

c. How many different pairs should there be? _____

The three questions

Ask these every single time, in this order.

1. Did I use **one fixed order**?
2. Did I check for **duplicates**?
3. Did I remember the **very last one**?

2 Try them out.

Sam listed every two-digit number using only the digits **1** and **2**. He wrote: 11, 12, 22.

a. Which question catches his mistake? _____

b. Write his list correctly.

_____

3 Two boxes, red or blue.

Each box gets painted red or blue. List every way to paint **two** boxes.

_____

Now how many ways for **three** boxes? _____ You may list them or reason it out.

✎ ADD TO YOUR COUNTING BOOK

Write the three questions on a fresh page. You will use them for the rest of the year, so leave room to add a fourth later.

💬 TALK ABOUT IT

Which of the three questions do you think catches the most mistakes? Why?

Is a list with a duplicate in it wrong, or just untidy? Say what you think and why.

STUCK? TRY THIS

For duplicates, read your list and cross off each pair on a rough drawing as you go. A pair you try to cross off twice is a duplicate.

★ CHALLENGE ZONE

Here is a list of every way to make 10 cents from pennies, nickels and dimes: one dime; two nickels; one nickel and five pennies; ten pennies. Is it complete? Prove your answer instead of saying it looks right.

Make up a list with exactly one duplicate and one gap, and hand it to somebody else to find.

Two Stages

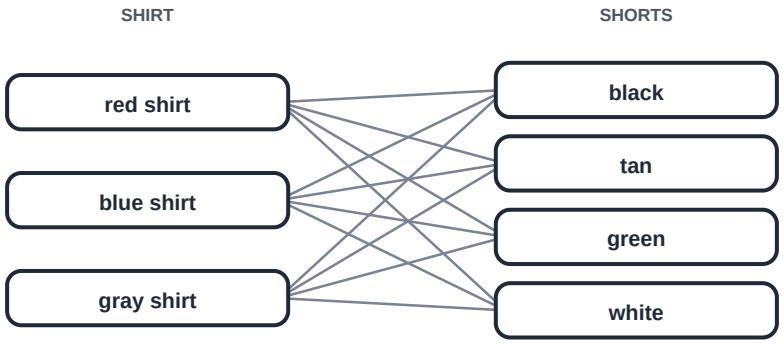
When two choices come one after the other, you multiply.

IN CLASS THIS WEEK
Equal groups and equal-group problems (3.OA.1, 3.OA.3)

NUMBER WARM-UP · 2 MINUTES
What is 4 groups of 5? What is 5 groups of 4?

WORDS TO KEEP
the multiplication rule if the first choice has 3 ways and the second has 4 ways, together they have 3 × 4 = 12 ways

Three shirts, four pairs of shorts. Every line below is one outfit.



1 Count it two ways.

a. Count the lines in the picture. How many? _____

b. Count the lines coming out of **one** shirt. _____ Now multiply that by the number of shirts. _____

c. Did both ways give the same answer? _____

2 Use the rule.

You do not have to draw the lines once you trust the rule.

Problem	first × second	answer
5 sandwiches, 3 drinks		
2 hats, 6 scarves		
7 colors, 4 sizes		
10 books, 3 bookmarks		

3 Which is more?

A shop sells **4** flavors of ice cream in **3** cone types. Another shop sells **6** flavors in **2** cone types.

a. How many different cones at each shop? _____ and _____

b. Explain why that happened, using the numbers.

_____

✎ ADD TO YOUR COUNTING BOOK

Write the multiplication rule in your own words, then draw your own two-stage picture for something in your house. Label both stages.

💬 TALK ABOUT IT

Explain to somebody why counting the lines and multiplying give the same answer. Use the picture, not the words *because it just does*.

Would the rule still work if one shirt could not be worn with the green shorts?

STUCK? TRY THIS

Cover the picture with your hand and look at only one shirt. Count its lines. Every shirt has exactly the same number, which is why multiplying works.

★ CHALLENGE ZONE

A lock has two dials. The first shows the letters A to E. The second shows the digits 0 to 9. How many different settings does the lock have?

A shop has 3 flavors and some number of cone types, and sells 21 different cones. How many cone types?

Rows and Columns Count

An array is not a picture of a count. It is the count.

IN CLASS THIS WEEK
Arrays and area models for multiplication (3.OA.3, 3.MD.7)

NUMBER WARM-UP · 2 MINUTES
What is 6 × 7? Draw it in your head as rows.

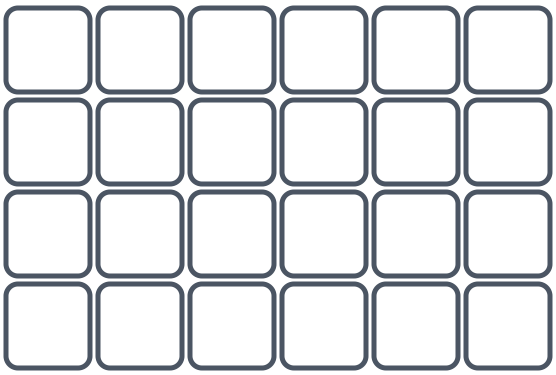
WORDS TO KEEP

array objects arranged in equal rows and equal columns

row goes across

column goes down

Here is a garden with **4 rows** of **6 plants**.



1 Three ways to the same number.

a. Count one row, then add: 6 + 6 + 6 + 6 = _____

b. Count one column, then add: 4 + 4 + 4 + 4 + 4 + 4 = _____

c. Multiply: 4 × 6 = _____

1 Turn the array.

A gardener turns the whole garden a quarter turn. Now it is **6 rows of 4**.

a. How many plants now? _____

b. Did anybody plant or pull up a single plant? Say what that tells you about 4 × 6 and 6 × 4.

_____

2 Last week's picture was an array.

Three shirts and four shorts made 12 outfits. Draw those 12 outfits as an array of dots, with one row per shirt.

How many rows did you draw? _____ How many in each row? _____

✎ ADD TO YOUR COUNTING BOOK

Draw one array and label it three ways: as repeated addition across, as repeated addition down, and as a multiplication. All three get the same number.

💬 TALK ABOUT IT

Why does turning an array not change how many things are in it?

Is every group of 24 things an array? Draw a group of 24 that is not.

STUCK? TRY THIS

Do not count the plants one at a time. Count **one row**, then count **how many rows**. Those are the only two numbers you need.

★ CHALLENGE ZONE

A theater has 12 rows of 15 seats. How many seats? If 8 seats are broken, how many still work?

Find every array you can make with exactly 12 dots. There are more than three. Write each as a multiplication.

Three Stages, One Tree

Two stages you can hold in your head. Three, you draw.

IN CLASS THIS WEEK
Properties of multiplication, multiplying three factors (3.OA.5)

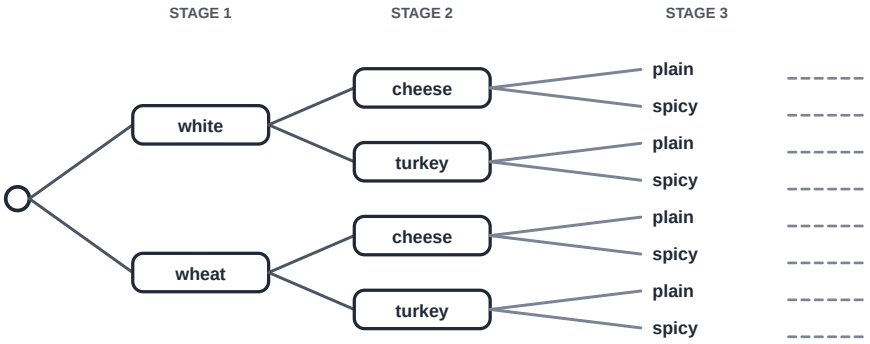
NUMBER WARM-UP · 2 MINUTES
What is 2 × 2 × 2? Now 2 × 3 × 2.

WORDS TO KEEP

tree diagram a picture where every path from left to right is one answer

branch one choice at one stage

A sandwich is **one bread**, then **one filling**, then **one sauce**. Follow any path from the dot on the left to a name on the right. That path is one sandwich.



1 Count the sandwiches.

a. Count the finished paths on the right. _____

b. Now multiply: 2 × 2 × 2 = _____

2 More stages, same rule.

Problem	multiply	answer
3 breads, 2 fillings, 2 sauces		
2 breads, 3 fillings, 4 sauces		
5 shirts, 2 shorts, 3 hats		
4 colors, 4 sizes, 2 shapes		

3 Draw one yourself.

Draw a tree for **2 drinks**, **3 snacks** and **2 fruits**. You do not have to write every name at the end, but you do have to draw every branch.

How many endings? _____

✎ **ADD TO YOUR COUNTING BOOK**

Copy your smallest tree onto a page and write beside it: **a tree with 2, then 3, then 2 branches has 2 × 3 × 2 = 12 endings.**

💬 **TALK ABOUT IT**

The tree gets very wide very fast. What does that tell you about adding one more stage?

Where in the tree can you see the number 2 × 2 without counting all the way to the right?

STUCK? TRY THIS

Every dot at one stage grows the **same** number of branches. Count the branches on one dot, not on all of them.

★ **CHALLENGE ZONE**

How many three-letter codes can you make from A, B and C if letters are allowed to repeat, so AAB and CCC both count?

Now how many if no letter may repeat? Explain why the second answer is so much smaller.

Order Matters

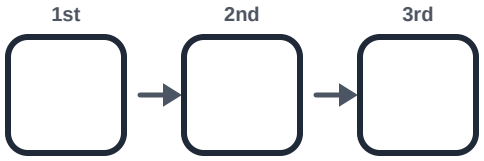
Gold, silver and bronze are not the same as just three winners.

IN CLASS THIS WEEK
Multiplying to solve problems, three factors (3.OA.5)

NUMBER WARM-UP · 2 MINUTES
What is 3 × 2 × 1? What is 4 × 3 × 2?

WORDS TO KEEP
arrangement a way of putting things in order. The long word for it is *permutation*

Three runners finish a race: **Ana**, **Ben** and **Cal**. Somebody has to get gold, somebody silver, somebody bronze.



gold, then silver, then bronze

1 Fill the slots one at a time.

a. How many runners could get gold? _____

b. Once gold is taken, how many are left for silver? _____

c. Then how many for bronze? _____

d. Multiply your three answers. _____

2 Write them all out.

List every finish, gold first. Start with Ana getting gold.

#	gold	silver	bronze
1			
2			
3			
4			
5			
6			

Does your list match your answer to part d? _____

3 Four runners, three medals.

a. Now **Dee** joins the race. How many could get gold? _____ Silver? _____ Bronze? _____

b. So how many different medal results? _____

✎ ADD TO YOUR COUNTING BOOK

Write: **when order matters, the choices shrink by one each time.** Under it put your three-runner list and your four-runner number.

💬 TALK ABOUT IT

Ana, Ben, Cal is a different result from Cal, Ben, Ana. Say why, in one sentence.

Would the answer change if the medals were three identical ribbons instead? Think hard before answering. Next week is about exactly this.

STUCK? TRY THIS

Do not try to list them and count. Fill the slots left to right and ask **how many are still available** at each slot.

★ CHALLENGE ZONE

Four books on a shelf. How many orders can you put them in?

Now five books.

How many arrangements are there of the three letters in **PIP**? The answer is not 6, and working out why is the whole puzzle.

Order Does Not Matter

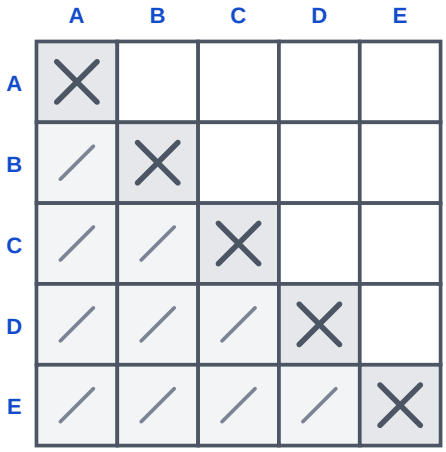
Picking a team is not the same as handing out medals.

IN CLASS THIS WEEK
Finding an unknown factor, dividing to share (3.OA.6, 3.OA.2)

NUMBER WARM-UP · 2 MINUTES
What is 20 ÷ 2? What is 12 ÷ 2?

WORDS TO KEEP
combination a choice where the order does **not** matter. Ana and Ben is the same team as Ben and Ana

Five friends: **A**, **B**, **C**, **D**, **E**. Two of them go on the trip. Every square below is a pair.



A crossed square is somebody paired with themselves. A shaded square is a pair you have already counted the other way round.

1 Read the picture.

a. How many squares altogether? 5 × 5 = _____

b. The crossed squares are pairs like **A with A**. How many are crossed? _____

c. Take those away. _____

d. The shaded half repeats the white half, because **B with A** is the same trip as **A with B**. So halve it. _____

2 The same count, without the grid.

Friends	× one fewer	÷ 2	pairs
4 friends	4 × 3 = 12		
5 friends			
6 friends			
8 friends			

3 Medals or team?

Write **M** if order matters and **T** if it does not.

a. Choosing 2 people to share a bench _____

b. Choosing a captain and a vice-captain _____

c. Choosing 2 toppings for one pizza _____

✎ ADD TO YOUR COUNTING BOOK

Write both rules on one page, side by side: **order matters** and **order does not matter**. Put your five-friend grid under the second one.

💬 TALK ABOUT IT

Why exactly does dividing by 2 fix the double counting? Point at two squares in the grid that are the same trip.

If three friends went instead of two, would you still divide by 2? Guess, then say why you guessed that.

STUCK? TRY THIS

The grid does the arguing for you. Every pair you want appears once in the white part and once again in the shaded part. That is the only reason you halve.

★ CHALLENGE ZONE

Ten friends, two go. How many pairs?

Choosing 2 people from 5 gives 10 answers. Arranging 2 people from 5 gives 20. Explain the 2 in between.

Handshakes, Bigger

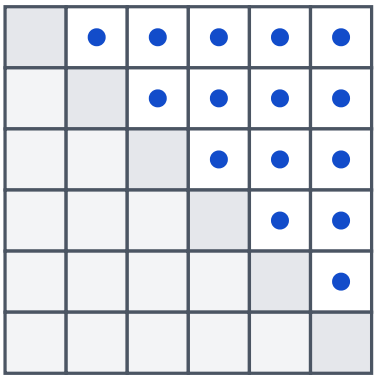
Everybody shakes hands with everybody, once. Now do it for a hundred people.

IN CLASS THIS WEEK
Fluency with multiplication and division within 100 (3.OA.7)

NUMBER WARM-UP · 2 MINUTES
What is 6 × 5? Now halve it.

WORDS TO KEEP
formula a rule written short, so it works for any number you put in

Six people at a party. Everybody shakes hands with everybody else exactly once. The grid counts it the same way last week's grid did.



all squares: 6 × 6 = 36

take away the 6 crossed

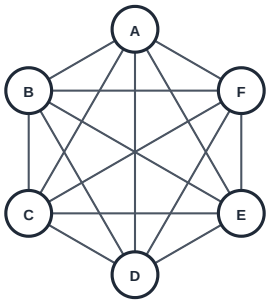
36 − 6 = 30

then halve, because each handshake is in there twice

30 ÷ 2 = 15

1 Check it against the picture.

Here are the same six people as dots, with every handshake drawn.



Count the lines. Does it match the grid? _____

2 Bigger parties.

People	n × (n − 1)	÷ 2	handshakes
8	8 × 7 = 56		
10			
12			
20			

3 Say the rule.

Fill it in: for **n** people, the number of handshakes is **n** times _____, then divided by _____.

✎ ADD TO YOUR COUNTING BOOK

Write the handshake rule on a page of its own. Beside it, draw the six-person grid and circle the part you divided in half. You will meet this rule again in Grade 6.

💬 TALK ABOUT IT

A hundred people at a party. Would you rather count the handshakes with the grid, the dots picture, or the rule? Say why.

Where does the “one fewer” come from? Point at it in the grid.

STUCK? TRY THIS

The two hard parts are separate. **One fewer** is because nobody shakes their own hand. **Halve** is because your handshake with Ana is Ana’s handshake with you.

★ CHALLENGE ZONE

A club had 21 handshakes. How many people were in the club? Work up from small numbers until you land on it.

How many handshakes at a party of 100? You may need to do the multiplication on paper.

WEEK 9 DOTS & LINES

Dots, Lines and Degree

How many lines meet at a dot, and what that number gives away.

IN CLASS THIS WEEK
Fluency with multiplication and division within 100 (3.OA.7)

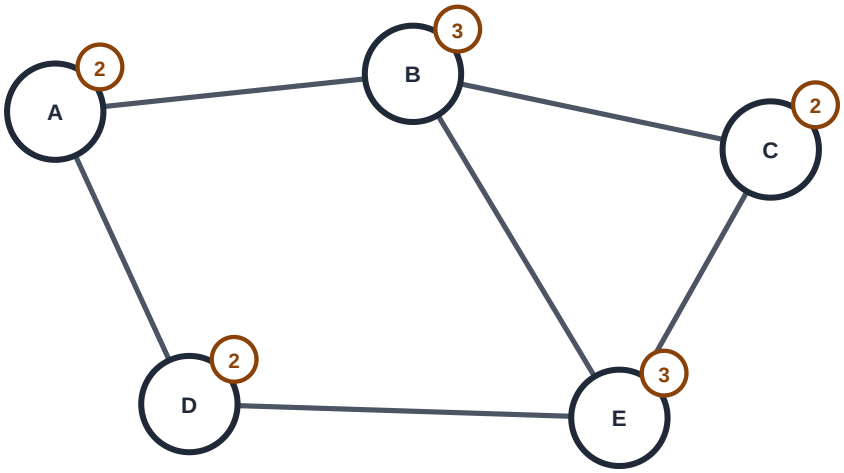
NUMBER WARM-UP · 2 MINUTES
What is 12 ÷ 2? What is 14 ÷ 2?

WORDS TO KEEP

degree how many lines meet at one dot

graph a drawing of dots joined by lines. It has nothing to do with bar graphs

Five friends. A line means those two know each other. The little circle by each dot is its **degree**, filled in for you.



1 Add them up.

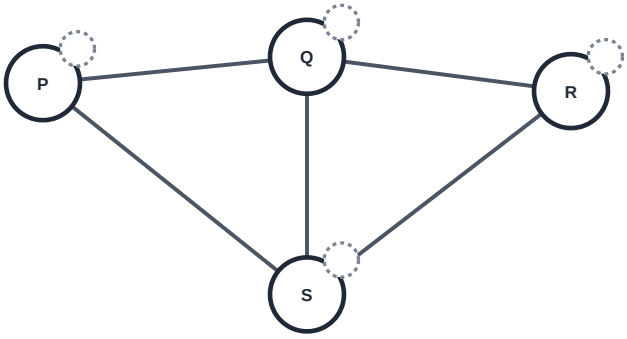
a. Add all five degrees. _____

b. Count the lines in the picture. _____

c. What do you notice about those two numbers?

_____

2 Your turn. Fill in the empty badges.



a. Sum of the degrees: _____

b. So the number of lines is _____. Check it against the picture.

3 Skip the picture.

a. A graph has 4 dots and every dot has degree 3. How many lines? _____

b. A graph has 10 lines. What is the sum of all its degrees? _____

✎ ADD TO YOUR COUNTING BOOK

Write: **every line adds 1 to the degree of each of its two dots, so the degrees always add up to twice the number of lines.** Draw a three-dot example that shows it.

💬 TALK ABOUT IT

Why does every single line count twice when you add up the degrees?

Could a dot have degree 0? Draw what that would look like.

STUCK? TRY THIS

Put your pencil on one line and follow it to both ends. You just added 1 to two different dots. Every line does exactly that.

★ CHALLENGE ZONE

Can you draw a graph with 5 dots where every dot has degree 3? Try it. If you cannot, the reason is more interesting than the drawing.

A party has 7 people and everybody shakes hands with exactly 4 others. How many handshakes? Use the degree rule, not a drawing.

WEEK 10 LOGIC & PROOF

Pigeons and Holes

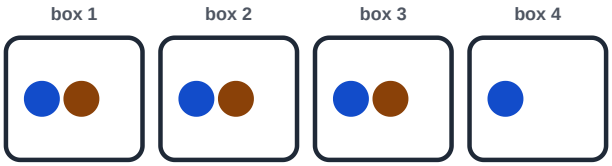
Sometimes you can be sure without looking.

IN CLASS THIS WEEK
Two-step word problems (3.OA.8)

NUMBER WARM-UP · 2 MINUTES
Seven things go into three boxes. Could every box hold exactly two?

WORDS TO KEEP
the pigeonhole principle if there are more things than boxes, then at least one box has to hold two or more. It does not matter which one

Seven marbles, four boxes. Somebody has already put them away, but you were not watching. Here is one way it could have gone.



7 things, 4 boxes

1 You do not need to see the boxes.

a. If every box held only one marble, how many marbles would that be? _____

b. But there are 7. So at least one box must hold at least _____ marbles.

c. Can you say **which** box? _____ Explain.

_____

2 Use the principle.

For each one, fill in the smallest number you can be **certain** about.

Situation	at least one has
13 socks, 12 drawers	
25 children, 12 birthday months	
10 letters, 9 mailboxes	
30 pencils, 7 cups	

3 Say why, not just what.

Pick the 25 children row. Write the reason a person who does not believe you would accept.

_____

_____

✎ **ADD TO YOUR COUNTING BOOK**

Write the pigeonhole principle in your own words. Then write the one thing it can **never** tell you: which box.

💬 **TALK ABOUT IT**

This is a proof about something nobody has looked at. Does that bother you? Say why it works anyway.

Could all seven marbles be in one box? Does that break the principle?

STUCK? TRY THIS

Ask the opposite question first: **what is the most you could fit if you spread things out as evenly as possible?** If there is still one left over, that one forces the answer.

★ **CHALLENGE ZONE**

In a class of 30, must two children share a birth month? Must three? Must four? Answer all three and say why the last one is different.

A human head has **fewer than 200,000** hairs, and more than a million people live in Los Angeles. Must two of them have exactly the same number of hairs? The word **fewer** is doing the work here. Say why the puzzle would fall apart without it.

WEEK 11 STEPS & RULES

Sorting Your Hand

Two people can both sort correctly and one can still do far less work.

IN CLASS THIS WEEK
Fluently adding and subtracting within 1000 (3.NBT.2)

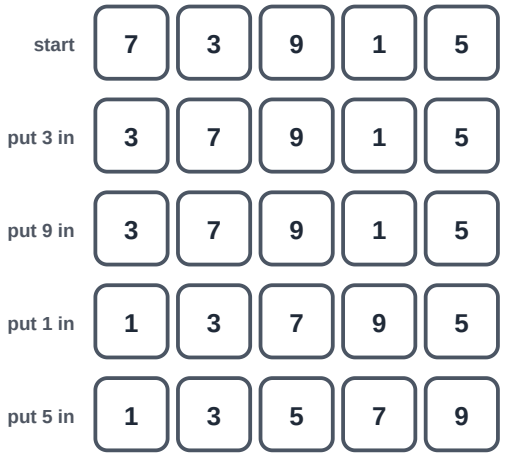
NUMBER WARM-UP · 2 MINUTES
Put these in order: 46, 64, 40, 60.

WORDS TO KEEP

algorithm steps written out so exactly that anybody following them gets the same answer

comparison one look at two cards to see which is bigger

Sorting a hand of cards: pick up the next card and slide it left until it is in the right place. Every card you **look at** on the way counts as one comparison.



1 Count the looks.

a. Putting **3** in, you looked at 7 and then ran off the end. That is _____ comparison.

b. Putting **1** in, you looked at 9, 7 and 3. That is _____.

c. Add up all four rows. Total comparisons: _____

2 Do it yourself.

Sort **4, 8, 2, 6** the same way. Write each row, then count.

_____

_____

_____

Total comparisons: _____

3 Easy hands and hard hands.

Same method, four cards, but the starting order is different.

Starting hand	comparisons
1, 2, 3, 4 (already sorted)	
4, 3, 2, 1 (backwards)	

Which starting hand is the most work, and why?

_____

✎ ADD TO YOUR COUNTING BOOK

Write down the sorting steps as an algorithm, numbered, so that somebody who has never seen it could follow them. Then write your best and worst comparison counts for four cards.

💬 TALK ABOUT IT

Both hands end up sorted. So why does it matter that one took more comparisons?

If a card is already in the right place, do you still have to look at anything? How many looks is the smallest possible?

STUCK? TRY THIS

Slide the new card left one place at a time and tap each card you pass. Count the taps, and remember to count the card that finally stops you.

★ CHALLENGE ZONE

Five cards already in order. How many comparisons? Now five cards exactly backwards. How many?

Look at your two answers and the four-card ones. Can you see the pattern in the backwards case? Write it down.

WEEK 12 STEPS & RULES

Splitting the Search

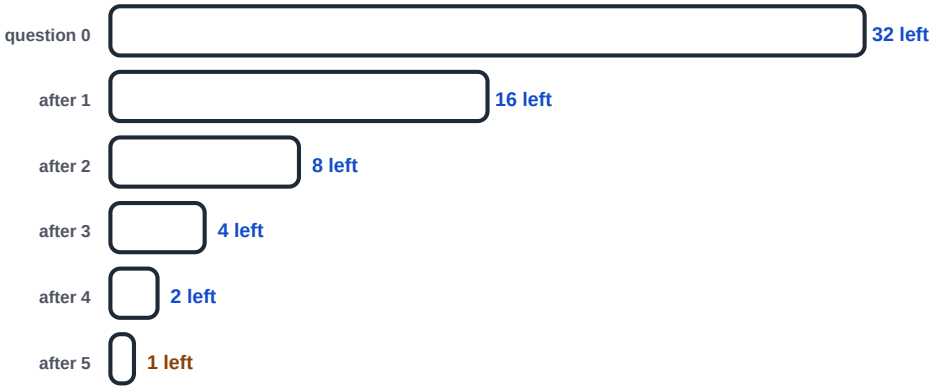
Guessing one at a time is the slowest possible plan.

IN CLASS THIS WEEK
Rounding to the nearest 10 or 100 (3.NBT.1)

NUMBER WARM-UP · 2 MINUTES
What is half of 32? Half of that? Keep going until you reach 1.

WORDS TO KEEP
halving cutting the group of possible answers in half with a single question

Pip is thinking of a number from **1 to 32**. You may only ask questions that get a yes or a no. Each bar below shows how many numbers are still possible.



1 Read the bars.

a. What question cuts 32 down to 16?

_____

b. How many questions until only one number is left? _____

2 How many questions?

Numbers to search	halvings needed
16	
32	5
64	
100	
1000	

The list gets ten times longer from 100 to 1000. Does the number of questions get ten times bigger? _____

3 The slow way.

a. If you guessed “is it 1? is it 2? is it 3?” one at a time, what is the **most** guesses you could need for 1 to 32? _____

b. And for 1 to 1000? _____

✎ **ADD TO YOUR COUNTING BOOK**

Write both methods on one page with their worst case for 1 to 1000 side by side. The gap between those two numbers is what this whole strand is about.

💬 **TALK ABOUT IT**

Halving only works if the numbers are in order. Why?

Is there a question that could cut 32 down to 8 in one go? Why not?

STUCK? TRY THIS

A good question splits what is left into two piles as close to equal as possible. Any question that leaves one pile much bigger is a wasted question.

★ **CHALLENGE ZONE**

Pip is thinking of a number from 1 to 1000. Write out the first three questions you would actually ask, with the range left after each one.

A librarian looks for one book among a million sorted books by halving. Roughly how many looks? The answer is smaller than almost anybody guesses.

WEEK 13 LOGIC & PROOF

Case Files

One case file. Everything from the first twelve weeks is in here somewhere.

IN CLASS THIS WEEK
Review and reasoning

NUMBER WARM-UP · 2 MINUTES
No warm-up this week. Read the whole page first, then start.

Five cases. You may solve them in any order, but each one needs a **reason** written next to the answer, not just the answer.

1 The lunch cart

The cart has 4 sandwiches, 3 drinks and 2 fruits. How many different lunches? Which week's rule did you use?

2 The incomplete list

Jo listed the ways to pick two different letters from A, B, C, D and got: AB, AC, AD, BC, BD. Is the list complete? If not, what is missing?

3 The busy dot

A graph has 6 dots. Every dot has degree 2. How many lines does it have?

4 The crowded shelf

A shelf holds 20 books in 6 stacks. What is the largest number you can be **certain** is in the tallest stack?

_____

5 The tidy hand

Sorting four cards takes 3 comparisons when they start in order and 6 when they start backwards. A hand takes 4. Is that possible? Give a hand that does it, or say why none exists.

_____

_____

Which tool?

Write the case number beside the tool it needed: multiplication rule _____, checking completeness _____, degree _____, pigeonhole _____, counting steps _____.

✎ **ADD TO YOUR COUNTING BOOK**

Add a page titled **My tools so far** and list all five, one line each, in the words you would use to explain them to somebody in your class.

💬 **TALK ABOUT IT**

Which case was hardest? What made it hard: the math, or working out which tool to use?

Pick any case and change one number in it. Does your method still work?

STUCK? TRY THIS

When you are stuck, do not reach for a method. Ask first: **does the order matter here, and am I counting choices or counting things?**

★ **CHALLENGE ZONE**

Invent a sixth case that needs two different tools at once. Write it out properly, with an answer key on a separate piece of paper.

WEEK 14 SHAPE & SPACE

Cutting a Rectangle

Area is an array that forgot it was an array.

IN CLASS THIS WEEK
Area by tiling, and area as multiplication (3.MD.6, 3.MD.7)

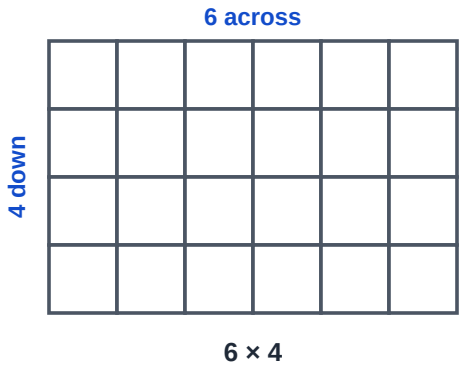
NUMBER WARM-UP · 2 MINUTES
What is 6 × 4? What is 6 × 2, twice?

WORDS TO KEEP

area how many unit squares it takes to cover a shape, with no gaps and no overlaps

square unit one of those squares

This rectangle is covered in unit squares.



6 × 4

1 Three ways again.

a. Count one row and add it 4 times. _____

b. Multiply across by down. _____

c. Now cut it into a 6 × 2 and another 6 × 2. Add those two areas. _____

2 Cut it where you like.

A 7 × 5 rectangle. Cutting it into a 7 × 3 and a 7 × 2 makes both pieces easy.

a. 7 × 3 = _____ and 7 × 2 = _____

b. Add them. _____

c. Check by multiplying 7 × 5 directly. _____

3 An awkward shape.

A room is shaped like an **L**. Draw any L made of squares, cut it into **two rectangles**, and find the total area.

Total area: _____ square units

✎ ADD TO YOUR COUNTING BOOK

Write: **a rectangle can be cut into pieces and the areas add up.** Draw your 6 × 4 cut two different ways to show it does not matter where you cut.

💬 TALK ABOUT IT

Why does cutting a rectangle up not change its area?

Would this still work if you cut it along a wiggly line instead of a straight one?

STUCK? TRY THIS

Cut along the lines that are already there. The pieces are rectangles too, and you already know how to find the area of a rectangle.

★ CHALLENGE ZONE

Find the area of a rectangle 9 across and 8 down by cutting it into a 9 × 4 and another 9 × 4. Then check it a different way.

Draw a shape that is not a rectangle and cannot be cut into rectangles at all.

WEEK 15 SHAPE & SPACE

Same Area, Different Shape

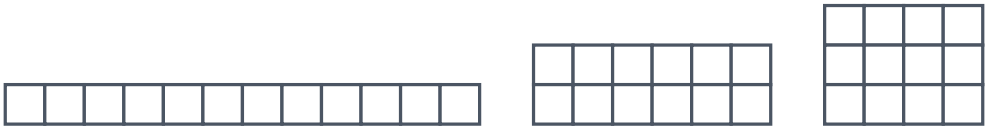
Twelve squares can be laid out several ways, and the fence changes every time.

IN CLASS THIS WEEK
Area and perimeter of rectangles (3.MD.7, 3.MD.8)

NUMBER WARM-UP · 2 MINUTES
What is 2 × (12 + 1)? What is 2 × (4 + 3)?

WORDS TO KEEP
perimeter the distance all the way around the outside

Every one of these rectangles is made of exactly **12** squares. They are the same area and different shapes.



12 × 1 6 × 2 4 × 3

1 Fill in the table.

Rectangle	area	perimeter
12 × 1	12	
6 × 2	12	
4 × 3	12	

Which shape needs the **least** fence? _____

2 Now with 16 squares.

Rectangle	area	perimeter
16 × 1		
8 × 2		
4 × 4		

Which one is a square? _____

3 Say what you found.

Out of all the rectangles with the same area, which shape uses the least fence? Write the rule you think is true.

_____

✎ ADD TO YOUR COUNTING BOOK

Draw all three of the 12-square rectangles side by side with their perimeters written under them. Circle the smallest perimeter and write why it is the squarest one.

💬 TALK ABOUT IT

A farmer with a fixed number of paving stones wants the least fence. What shape should the patio be?

Is 12 × 1 really a different rectangle from 1 × 12, or the same one turned?

STUCK? TRY THIS

Perimeter is across plus down, then doubled. Long thin rectangles have a big *across*, so they cost a lot of fence.

★ CHALLENGE ZONE

Find every rectangle with an area of 36 squares and work out each perimeter. Which is smallest?

Is there an area where **no** square rectangle exists? Find one and say what is special about that number.

WEEK 16 SHAPE & SPACE

Same Perimeter, Different Area

Now hold the fence still and see what happens to the ground.

IN CLASS THIS WEEK
Perimeter, and rectangles with the same perimeter (3.MD.8)

NUMBER WARM-UP · 2 MINUTES
Two numbers add to 8. Name three different pairs.

WORDS TO KEEP
the same perimeter if across plus down is the same, the perimeter is the same, however different the two rectangles look

Each of these has a perimeter of exactly **16**. The across and the down always add to 8.

1 Fill in the table.

Rectangle	across + down	perimeter	area
7 × 1	8	16	
6 × 2	8	16	
5 × 3	8	16	
4 × 4	8	16	

Which one holds the **most**? _____

2 Twenty meters of fence.

A pen has a perimeter of 20 meters, so across plus down is 10.

across × down	area
1 × 9	
2 × 8	
3 × 7	
4 × 6	
5 × 5	

Biggest area: _____ square meters

3 Both rules together.

Last week the square used the **least** fence. This week the square holds the **most** ground. Are those two different facts or the same fact said twice?

_____

_____

✎ **ADD TO YOUR COUNTING BOOK**

Put last week's page and this week's page next to each other. Write one sentence at the bottom of both that covers them together.

💬 **TALK ABOUT IT**

Why does a long thin pen waste so much ground?

If the pen could be any shape at all, not just a rectangle, what shape would hold the most? Guess, then say why you guessed it.

STUCK? TRY THIS

Every rectangle here has the same across-plus-down. Pick pairs that add to your number, then multiply each pair to get its area.

★ **CHALLENGE ZONE**

A farmer has 24 meters of fence. What is the largest rectangular pen, and what are its sides?

The pen is now built against a wall, so only three sides need fence. With 24 meters, what is the largest pen now? This changes the answer.

WEEK 17 WAYS OF COUNTING

Rectangles Inside a Grid

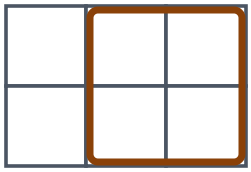
How many rectangles are hiding in a grid? More than you think.

IN CLASS THIS WEEK
Multiplication and division problems (3.OA.3)

NUMBER WARM-UP · 2 MINUTES
How many 1 × 1 squares are in a 3 across by 2 down grid?

WORDS TO KEEP
systematic the same word as Week 1, and the same job: count by size, in order, so nothing is skipped

A grid **3 across** and **2 down**. One rectangle inside it has been outlined. Your job is to count **every** rectangle, of every size.



1 Count them by size.

Size	how many
1 × 1	6
2 × 1	
3 × 1	
1 × 2	
2 × 2	
3 × 2	

Total: _____

2 A smaller grid.

Now a grid **2 across** and **2 down**. Count by size again.

Size	how many
1 × 1	
2 × 1	
1 × 2	
2 × 2	

Total: _____

3 A bigger one.

A grid **4 across** and **2 down**. Use the same table on scrap paper. Total: _____

✎ ADD TO YOUR COUNTING BOOK

Write the counting-by-size method down as a numbered list of steps. It is the Week 1 idea again, and saying that out loud is the point.

💬 TALK ABOUT IT

Why is counting by size better than hunting for rectangles at random?

Which size did you almost forget? Nearly everybody forgets the same one.

STUCK? TRY THIS

Go size by size in a fixed order: all the 1 wide, then all the 2 wide, then all the 3 wide. Inside each width, do 1 tall before 2 tall. Do not skip the whole grid itself.

★ CHALLENGE ZONE

How many rectangles in a 3 across by 3 down grid?

Look at your totals for 2 by 2, 3 by 2 and 4 by 2. There is a pattern in them. Find it and predict 5 by 2 before you count.

Factor Pairs

Every way to build a rectangle out of 12 squares is a fact about the number 12.

IN CLASS THIS WEEK
Finding an unknown factor; factors and products (3.OA.6, 3.OA.4)

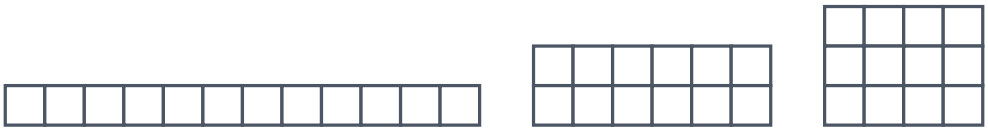
NUMBER WARM-UP · 2 MINUTES
What times 3 makes 12? What times 4 makes 12?

WORDS TO KEEP

factor a number that divides another one exactly, with nothing left over

factor pair two factors that multiply to make the number

Every rectangle you can build from exactly **12** squares. You met these in Week 15, but this time look at the numbers under them, not the shapes.



12 × 1 6 × 2 4 × 3

1 From rectangles to factors.

a. Write the three factor pairs of 12.

_____

b. Now list every factor of 12, smallest first.

_____

c. How many factors does 12 have? _____

2 Other numbers.

Number	factor pairs	all its factors	how many
18			
24			
7			
16			

3 Two odd ones out.

a. Which number in your table has only **two** factors? _____

b. Which one can be built as a **square** rectangle? _____ Count its factors again. Something odd is going on with that count.

✎ **ADD TO YOUR COUNTING BOOK**

Draw the factor rectangles of 12 and of 16 on one page. Under each, write its factor list. Circle the square one.

💬 **TALK ABOUT IT**

Why does every factor pair come with a rectangle, and every rectangle with a factor pair?

Why does a square rectangle only give you **one** new factor instead of two?

STUCK? TRY THIS

Try dividing your number by 1, then 2, then 3, and so on. Every time it divides exactly, you have found a factor pair. You can stop once the pairs start repeating.

★ **CHALLENGE ZONE**

Find every number from 1 to 20 that has exactly **two** factors. There are eight of them. Next week is about what they are called.

Find every number from 1 to 20 that has an **odd** number of factors. They all have something in common.

Only Two Rectangles

Some numbers can only be built one way, and that turns out to matter.

IN CLASS THIS WEEK
Multiplication and division fluency (3.OA.7)

NUMBER WARM-UP · 2 MINUTES
Which of these divide 12 exactly: 2, 4, 5, 6?

WORDS TO KEEP

prime a number with exactly two factors, 1 and itself

composite a number with more than two factors

neither 1 has only one factor, so it is not prime and not composite

Three numbers, and every rectangle each one can make. Look at how many each number gets.

7 × 1

9 × 1

3 × 3

1 Sort them.

a. How many rectangles can 7 make? _____ How many factors does 7 have? _____

b. Same two questions for 9. _____ _____

c. Which of them is **prime**? _____

2 Hunt for primes up to 30.

Cross out every number that has a rectangle other than the long thin one. What is left is prime.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30						

List the primes up to 30.

How many are there? _____

3 The one that is neither.

Is 1 prime? Count its factors first, then read the definition again, then answer.

✎ ADD TO YOUR COUNTING BOOK

Add a page called **Primes to 30**. Write them out. You will use this list in Grade 4 and again in Grade 6, so make it neat.

💬 TALK ABOUT IT

Are there more primes below 15 or between 15 and 30? Why might that be?

Is 2 prime? It is the only even one, which makes people doubt it. Check it properly.

STUCK? TRY THIS

You do not have to test every possible factor. Once you have tried up to the point where the pairs start repeating, you are done.

★ CHALLENGE ZONE

Is 51 prime? It looks prime and it is not. Find the factor pair.

Find two primes that add up to 20. Then find another pair. Then try 30.

Sharing Without Remainder

Division asks two different questions, and they are not the same question.

IN CLASS THIS WEEK
Division as equal groups and equal shares (3.OA.2)

NUMBER WARM-UP · 2 MINUTES
24 shared among 4. Now 24 in groups of 4. Are those the same?

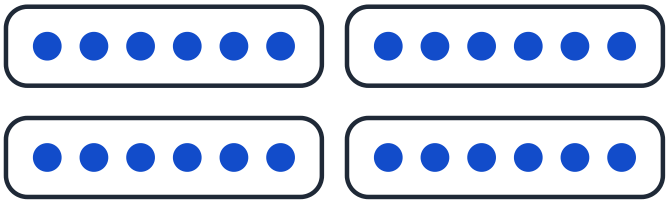
WORDS TO KEEP

quotient the answer to a division

how many groups 24 ÷ 6 can mean: how many groups of 6 fit in 24?

how many each or it can mean: 24 shared among 6, how many each?

Twenty-four counters, put into groups of **six**.



1 The same numbers, two questions.

a. How many groups of 6 fit in 24? _____

b. If 24 is shared among 4 people, how many each? _____

c. The answers are different questions with the same number. Draw the second one and see how the picture changes.

2 Exact or not.

Put a tick if the division comes out exactly, with nothing left over.

Division	exact?	answer if exact
36 ÷ 4		
36 ÷ 5		
36 ÷ 6		
36 ÷ 7		
36 ÷ 8		
36 ÷ 9		

3 What the ticks mean.

The numbers that divided 36 exactly have a name you learned two weeks ago. What is it?

_____

✎ ADD TO YOUR COUNTING BOOK

Write both meanings of division on one page, each with its own drawing of 24 ÷ 6. Two pictures, one number sentence.

💬 TALK ABOUT IT

Which of the two questions do you picture when somebody says “divide”? Ask somebody else which one they picture.

Why does a division come out exactly only when the number you divide by is a factor?

STUCK? TRY THIS

Draw it. Groups of 6 means rings of six counters. Shared among 6 means six piles that grow one counter at a time.

★ CHALLENGE ZONE

36 cookies go into equal bags with none left over. There must be at least 2 bags and at least 2 cookies per bag. How many different bag counts work?

Find a number under 50 that can be shared exactly among 2, 3, 4 and 6 people. Then find every other one.

WEEK 21 NUMBER STRUCTURE

What Is Left Over

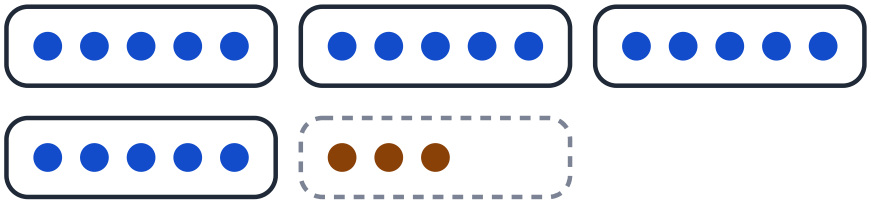
The leftovers are not a mistake. They repeat, and the repeating is useful.

IN CLASS THIS WEEK
Two-step problems and interpreting remainders (3.OA.8)

NUMBER WARM-UP · 2 MINUTES
What is 23 ÷ 5? Is there anything left over?

WORDS TO KEEP
remainder what is left when a division does not come out exactly. It is always smaller than the number you divided by

Twenty-three counters in groups of **five**. The dashed ring is what would not fill a whole group.



1 Read it off.

a. Full groups: _____ Left over: _____

b. Write it: 23 ÷ 5 = _____ remainder _____

c. Could the remainder ever be 5? _____ Say why.

_____

2 Practice.

Division	quotient	remainder
17 ÷ 5		
29 ÷ 4		
30 ÷ 7		
100 ÷ 9		

3 The remainders repeat.

Divide each number by **4** and write only the remainder.

number	1	2	3	4	5	6	7	8	9	10	11	12
remainder	1	2	3	0								

How long is the pattern before it repeats? _____

✎ ADD TO YOUR COUNTING BOOK

Write the remainder pattern for dividing by 4 on a page, and under it the same pattern for dividing by 3. Both of them loop. Say how long each loop is.

💬 TALK ABOUT IT

Why can the remainder never be as big as the number you are dividing by?

If you keep counting past 12, can you say the remainder for 20 without dividing?

STUCK? TRY THIS

Take away whole groups until what is left is too small to make another group. That is the remainder, and it can be 0.

★ CHALLENGE ZONE

What is the remainder when 100 is divided by 3?

Now 1000 divided by 3, without doing the long division. Look at 999 first.

WEEK 22 NUMBER STRUCTURE

The Days Wrap Around

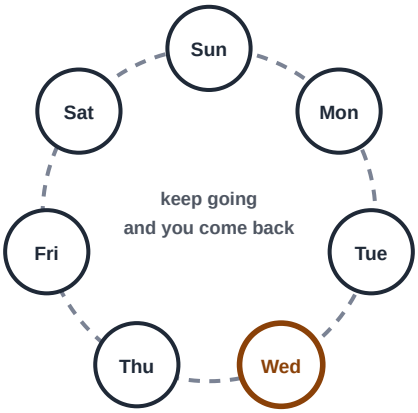
Thirty days from Monday. You can work it out without counting thirty days.

IN CLASS THIS WEEK
Solving two-step problems; telling time and elapsed time (3.OA.8, 3.MD.1)

NUMBER WARM-UP · 2 MINUTES
What is 30 ÷ 7? What is the remainder?

WORDS TO KEEP
wrapping around when counting runs off the end and starts again at the beginning, the way days of the week do

The days go around a ring. After Saturday there is no eighth day, you land back on Sunday. Wednesday is marked.



1 Thirty days from Monday.

a. How many whole weeks fit in 30 days? _____ With how many days left over? _____

b. Whole weeks bring you back to the same day, so only the leftover matters. Move _____ days on from Monday.

c. The answer is _____

2 Do four more.

Start day	days later	÷ 7 leaves	lands on
Tuesday	10		
Sunday	100		
Thursday	365		
Friday	49		

3 The one that does not move.

One row above lands on the day it started. Which one, and what is special about its number of days?

✎ **ADD TO YOUR COUNTING BOOK**

Write: **only the remainder after dividing by 7 changes the day.** Draw the ring and put your 30-days-from-Monday answer on it.

💬 **TALK ABOUT IT**

Why do the whole weeks not matter at all?

Clocks wrap around too. What number do they wrap at? What about a 24-hour clock?

STUCK? TRY THIS

Divide the number of days by 7 and throw the whole weeks away. Only the remainder moves you around the ring.

★ **CHALLENGE ZONE**

Your birthday is on a Saturday this year. What day will it be next year, if there is no February 29 in between? A normal year is 365 days.

And if there **is** a February 29 in between, so 366 days? Explain why one extra day changes the answer.

WEEK 23 DATA

Reading a Scaled Graph

One square does not always mean one thing, and that is where mistakes live.

IN CLASS THIS WEEK
Scaled bar graphs and picture graphs (3.MD.3)

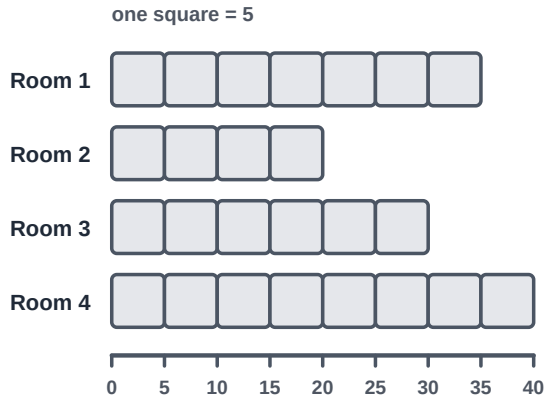
NUMBER WARM-UP · 2 MINUTES
What is 7 × 5? What is 4 × 5?

WORDS TO KEEP

scale how much one square on the graph is worth

key the note that tells you the scale. Read it first, always

Books read by each class. Read the key before you read the bars.



1 Read it.

a. How many books did Room 1 read? _____

b. How many more did Room 4 read than Room 2? _____

c. How many books in all four rooms? _____

2 A different scale.

The same four rooms, drawn again with **one square = 10**. How many squares long is each bar now?

Room	books	squares at 10 each
Room 1	35	
Room 2	20	
Room 3	30	
Room 4	40	

One of them will not come out to a whole number of squares. Which? _____

3 Why bother with a scale?

If one square meant one book, how many squares long would Room 4's bar be? _____
Would that fit on this page?_____

✎ ADD TO YOUR COUNTING BOOK

Draw the four-room graph twice on one page, once at a scale of 5 and once at a scale of 10. Write under both: **same data, different picture**.

💬 TALK ABOUT IT

Two graphs of the same numbers can look completely different. Is that dishonest, or just a choice?

What scale would you pick for a graph about the number of people in four cities?

STUCK? TRY THIS

Multiply, do not count. Count the squares in one bar, then multiply by whatever the key says one square is worth.

★ CHALLENGE ZONE

A graph has a scale of 5. Somebody wants to show 34 books. Can they? Say exactly what goes wrong.

Design a scale that would let you show 35, 20, 30 and 40 all as whole squares. There is more than one right answer.

WEEK 24 DATA

Counting From a Picture

Half a symbol is where nearly everybody gets it wrong.

IN CLASS THIS WEEK
Picture graphs with a scaled key (3.MD.3)

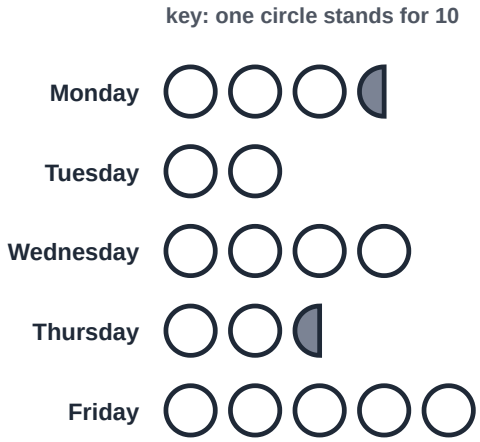
NUMBER WARM-UP · 2 MINUTES
What is half of 10? What is 3 × 10, plus half of 10?

WORDS TO KEEP

pictograph a graph where a picture stands for several things

half symbol half a picture means half of whatever the key says

Lunches sold last week. One circle stands for 10 lunches.



1 Read it carefully.

a. Monday: _____ lunches b. Thursday: _____

c. Which day sold the most? _____

d. How many lunches all week? _____

2 What a pictograph cannot show.

a. With a key of 10 and only whole and half circles, could you show **27** lunches? _____

b. What is the closest you could get? _____

c. What key would let you show 27 exactly? _____

3 Build your own.

Draw a pictograph for these, choosing your own key: 18, 12, 24 and 6.

My key: one circle = _____

✎ ADD TO YOUR COUNTING BOOK

Write down the rule for picking a key: **the key should be a factor of every number you need to show.** Try it against your own graph above.

💬 TALK ABOUT IT

Why is a key of 6 a good choice for 18, 12, 24 and 6?

Somebody draws a quarter of a circle. Is that fair? What would it be worth?

STUCK? TRY THIS

Read the key first and write it at the top of your working. Then count whole circles, multiply, and add half the key for a half circle.

★ CHALLENGE ZONE

A pictograph has a key of 6 and one row shows two and a half circles. How many is that?

Could that same graph show exactly 20? Say what stops it.

WEEK 25 LOGIC & PROOF

Puzzle Fair

A little of everything from Weeks 14 to 24. No new rules this week.

IN CLASS THIS WEEK
Review and reasoning

NUMBER WARM-UP · 2 MINUTES
No warm-up. Pick whichever puzzle looks best and start there.

Six puzzles. Each one needs an answer **and** the name of the tool you used. Some need two tools.

1 The stubborn fence

A rectangular pen has an area of 24 square meters. Which shape uses the least fence? List every rectangle first.

2 The hidden rectangles

How many rectangles of every size are inside a grid 3 across and 3 down?

3 The awkward number

Is 91 prime? Do not guess. Show your test.

4 The long wait

Today is Wednesday. What day is it in 100 days?

5 The bad key

A pictograph must show 14, 21 and 35. What key works, and why does a key of 10 fail?

6 The leftover

A teacher puts 100 pencils into cups of 7. How many full cups, and how many pencils are left?

✎ ADD TO YOUR COUNTING BOOK

Add a page called **My tools, weeks 14 to 24**. Area and perimeter, factors, primes, remainders, wrapping around, scales and keys. One line each, in your own words.

💬 TALK ABOUT IT

Which puzzle needed two tools at once? Which two?

Puzzle 3 and puzzle 6 both use division, but for completely different reasons. Say what each one is really asking.

STUCK? TRY THIS

If a puzzle looks like nothing you have seen, ask what it is really counting. Almost everything this trimester is either a rectangle, a factor, or a leftover.

★ CHALLENGE ZONE

Write a seventh puzzle that uses the wrapping-around idea and the remainder idea together. Include an answer key on a separate sheet.

WEEK 26 FRACTIONS

Parts of a Whole

Equal parts. The word equal is doing all the work in that sentence.

IN CLASS THIS WEEK
Understanding fractions as equal parts of a whole (3.NF.1)

NUMBER WARM-UP · 2 MINUTES
If a strip is cut into 4 equal parts, what is one part called?

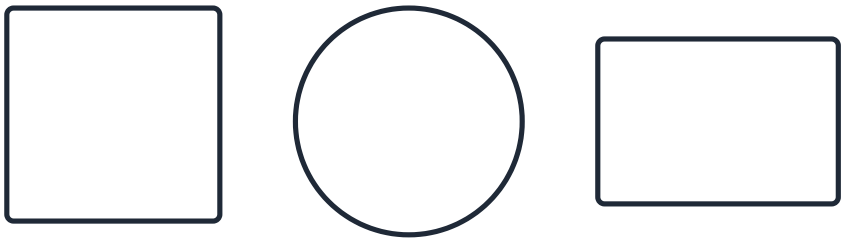
WORDS TO KEEP

denominator the bottom number: how many equal parts the whole was cut into

numerator the top number: how many of those parts you have

unit fraction one single part, like one fourth

Cut each shape into **four equal parts** and shade one of them. There is more than one right way for each.



1 Name what you shaded.

a. Each shaded piece is one _____ of the whole.

b. Written as a fraction: _____

c. If you shaded three pieces instead, what fraction? _____

2 Equal or not?

Somebody cut a square into four parts, but one part is clearly bigger than the rest. Can they call one part a fourth?

Answer, and say what word in the definition settles it.

_____

3 Read the fractions.

Cut into	you take	fraction	read it out loud as
8 parts	3		
6 parts	1		
3 parts	2		
10 parts	7		

✎ **ADD TO YOUR COUNTING BOOK**

Start a fractions page. Draw one whole cut into halves, thirds, fourths and sixths, all the same length so they line up under each other. You will use this strip next week.

💬 **TALK ABOUT IT**

Two people cut the same cake into four parts and their pieces look different shapes. Can both be right?

Why is one sixth smaller than one third, when 6 is bigger than 3?

STUCK? TRY THIS

Count the parts the whole was cut into first. That number goes on the bottom, every time, before you look at how many you took.

★ **CHALLENGE ZONE**

Cut a square into four equal parts in three genuinely different ways. Shapes that are the same one turned around do not count as different.

Can you cut a square into four equal parts where none of them is a square or a rectangle?

Making One Whole

Counting, but with fraction pieces. Same job: list them in order, miss none.

IN CLASS THIS WEEK
Fractions on a strip; equivalent amounts (3.NF.1, 3.NF.3)

NUMBER WARM-UP · 2 MINUTES
How many sixths make one third? How many sixths make one half?

WORDS TO KEEP

piece here it means one half, one third or one sixth

a way one collection of pieces that adds to exactly one whole

You have as many **halves**, **thirds** and **sixths** as you like. Make exactly **one whole**. Using six sixths is one way. Using one half and three sixths is another.

The same whole four times. Read straight down to see how many sixths each piece is worth.

1 Get organized first.

a. How many sixths in one half? _____ In one third? _____

b. So one whole is _____ sixths altogether.

c. Now every way is really a way of making that number out of 3s, 2s and 1s.

2 List them in order.

Start with the ways that use **two** halves, then **one** half, then **none**. Inside each, use as many thirds as you can first.

#	halves	thirds	sixths
1			
2			
3			
4			
5			
6			
7			

3 Count them.

a. How many ways altogether? _____

b. Which of the three questions from Week 2 did you use to be sure?

_____

✎ **ADD TO YOUR COUNTING BOOK**

Copy your finished list onto the fractions page from last week. Write beside it: **the same fixed order that worked for lunches works for fractions.**

💬 **TALK ABOUT IT**

Why was turning everything into sixths the move that made this easy?

If quarters were allowed as well, would the list get longer? Do not count it, just say why.

STUCK? TRY THIS

Turn every piece into sixths. One half is 3 sixths, one third is 2 sixths. Now you are just making 6 out of 3s, 2s and 1s, which is a counting problem you have done before.

★ **CHALLENGE ZONE**

Do the whole thing again allowing **quarters** instead of thirds, so halves, quarters and eighths. Turn everything into eighths first.

Which is the shorter list, and why does the number of pieces in the whole decide it?

WEEK 28 FRACTIONS

Same Amount, Different Name

One half and three sixths are the same amount wearing different clothes.

IN CLASS THIS WEEK
Equivalent fractions and comparing fractions (3.NF.3)

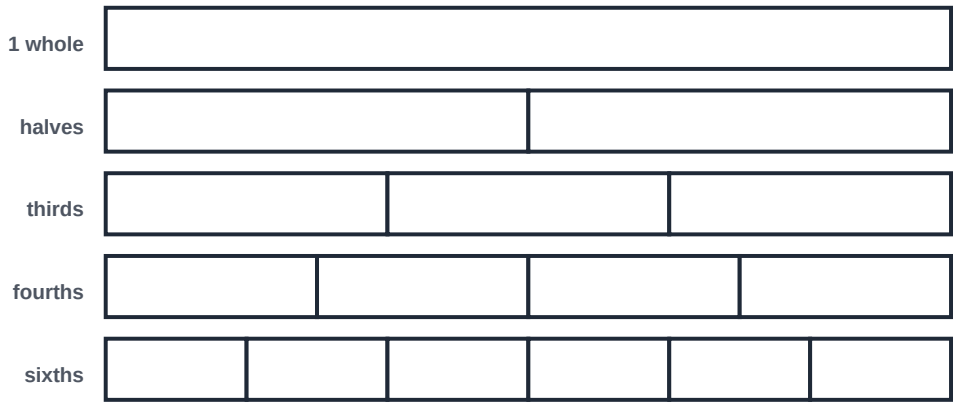
NUMBER WARM-UP · 2 MINUTES
How many fourths make one half? How many sixths make one half?

WORDS TO KEEP

equivalent two fractions that are the same amount, written differently

compare decide which is bigger. Only fair when the wholes are the same size

Every strip below is the **same whole**, cut into a different number of equal parts. Line them up with your finger.



1 Read straight down.

a. One half lines up with _____ fourths and _____ sixths.

b. Two thirds lines up with _____ sixths.

c. Does one third line up with any number of fourths? _____ Look carefully before you answer.

2 Same bottom number.

When the wholes are cut the same way, the bigger top number wins. Write < or >.

3 eighths _____ 5 eighths 4 sixths _____ 1 sixth

Say why that rule is safe.

_____

3 Same top number.

Now the tops match and the bottoms do not. Write < or >.

2 thirds _____ 2 fifths 3 fourths _____ 3 eighths

This one catches people out. Why does a **bigger** bottom number make a **smaller** fraction?

_____

✎ ADD TO YOUR COUNTING BOOK

Add the strip picture to your fractions page and mark every place two strips line up. Each line-up is a pair of equivalent fractions. Write them all out.

💬 TALK ABOUT IT

Cutting a cake into more pieces does not give you more cake. Say what it does give you.

Would comparing 2 thirds and 3 fourths be safe with the same-top or same-bottom rule? Neither one fits, which is a Grade 4 problem.

STUCK? TRY THIS

Put your finger on the strip and slide it straight down. Two marks that line up exactly are the same amount, whatever they are called.

★ CHALLENGE ZONE

Find three different names for the amount one half, using the strips you have.

Draw a strip cut into twelfths. How many new line-ups does it give you?

WEEK 29 DOTS & LINES

Coloring a Schedule

Five clubs, and some of them cannot meet at the same time.

IN CLASS THIS WEEK
Two-step problems and reasoning (3.OA.8)

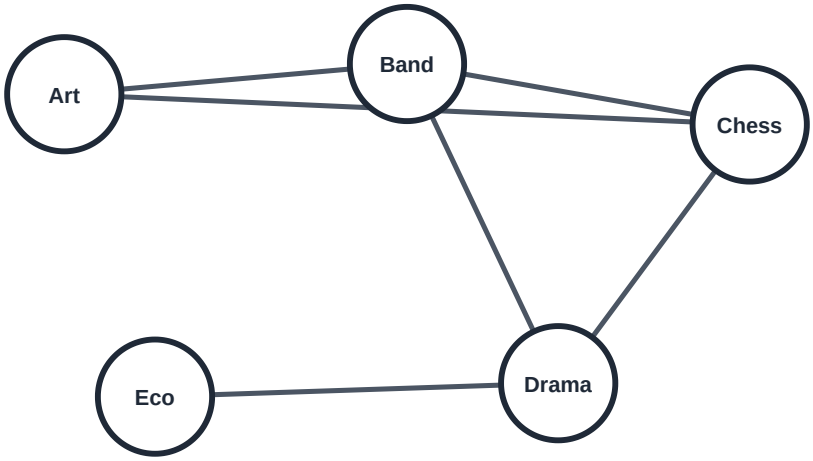
NUMBER WARM-UP · 2 MINUTES
If three things all clash with each other, how many time slots do they need?

WORDS TO KEEP

conflict two clubs that share a member, so they cannot meet at the same time

time slot a period when clubs meet. Clubs with no line between them may share one

A line between two clubs means at least one student is in both, so they **cannot** meet at the same time.



1 Find the hard part first.

a. Find three clubs that **all** clash with each other.

_____

b. Those three alone need how many different time slots? _____ So the answer cannot be smaller than that.

2 Now schedule them.

Give every club a slot number. Two clubs joined by a line must get **different** numbers. Use as few slots as you can.

Club	slot
Art	
Band	
Chess	
Drama	
Eco	

Slots used: _____ Check every line before you say you are done.

3 Which clubs share a slot?

Name two clubs that meet at the same time in your answer, and say why that is allowed.

_____

✎ **ADD TO YOUR COUNTING BOOK**

Draw the club graph and color it with your slots. Write underneath: **three clubs that all clash need three slots, so three was the smallest possible answer.**

💬 **TALK ABOUT IT**

You proved the answer could not be less than 3, and then you found a way to do it in 3. Why do you need **both** of those?

If one more student joined Art and Eco, would the answer change?

STUCK? TRY THIS

Start with the busiest cluster, not with the first club in the list. Once the clubs that all clash are placed, the rest usually drop into slots you have already used.

★ **CHALLENGE ZONE**

Add a sixth club that clashes with all five. How many slots now?

Draw a graph of five clubs that needs only **two** slots. What has to be true about it?

WEEK 30 DOTS & LINES

Can Everyone Be Paired?

Four students, four jobs. Sometimes it fits and sometimes nothing you try will.

IN CLASS THIS WEEK
Two-step problems and reasoning (3.OA.8)

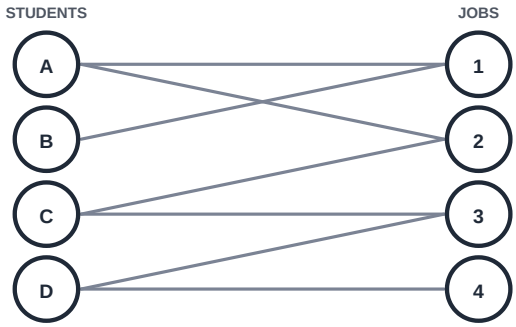
NUMBER WARM-UP · 2 MINUTES
Three people all want the same one job. How many of them can have it?

WORDS TO KEEP

matching giving every student a job, with no job used twice

possible here it means: there is a way, and you can show it

A line means that student is able to do that job. Give **everybody** a job, with no job done twice.

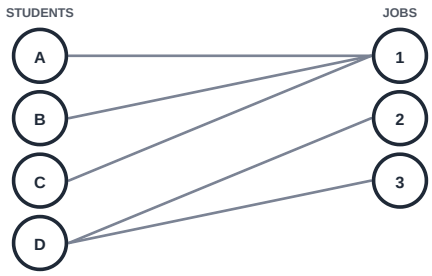


1 Find the matching.

a. Which student has only **one** job they can do? _____ Give them that job first.

b. Now finish it. A: _____ B: _____ C: _____ D: _____

2 Now this one.



a. Try to give everybody a job. How many students end up with one? _____

b. Which three students are the problem, and what do they have in common?

_____

3 You have seen this before.

Three students, one job between them. Which rule from Week 10 does that?

_____

✎ **ADD TO YOUR COUNTING BOOK**

Write the rule you found: **if some group of students can only reach fewer jobs than there are of them, no matching exists.** Draw the failing picture beside it.

💬 **TALK ABOUT IT**

In the second picture, is the problem that you tried the wrong order? Prove it is not.

Does having the same number of students and jobs guarantee it will work?

STUCK? TRY THIS

Start with whoever has the fewest choices. If somebody can only do one job, that job is settled before you touch anything else.

★ **CHALLENGE ZONE**

Draw four students and four jobs where it fails, but where every student can do at least **two** jobs. This is harder than it sounds.

Five students, five jobs, and it works. What is the smallest number of lines you can draw?

WEEK 31 STEPS & RULES

Instructions With a Repeat

The same four sides, written once instead of four times.

IN CLASS THIS WEEK
Patterns in arithmetic; two-step problems (3.OA.9, 3.OA.8)

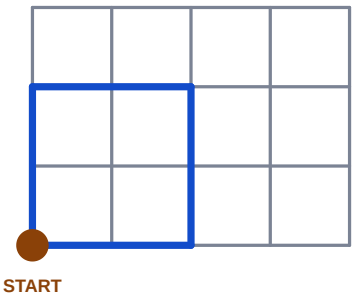
NUMBER WARM-UP · 2 MINUTES
What is 3 × 4? If you repeat 3 steps four times, how many steps in all?

WORDS TO KEEP

repeat block an instruction that says: do these steps again, this many times

loop another name for the same thing

A robot starts at the dot and follows instructions. **F** means go forward one square. **R** means turn right without moving.



1 Write it the long way.

a. Write the whole program for that square path using only F and R.

_____

_____

b. How many instructions did that take? _____

2 Now the short way.

The same path, written with a repeat:

REPEAT 4 TIMES [F F R]

a. How many instructions are written down now? _____

b. How many does the robot actually carry out? _____

3 A path that does not fit one repeat.

Now the robot must trace a rectangle **4 across and 2 up**. The four sides are not all the same length.

a. Write it with a repeat that runs **twice**.

_____

b. How many instructions written, and how many carried out? _____ and _____

✎ ADD TO YOUR COUNTING BOOK

Write both versions of the square program on one page, side by side, with the number of written instructions under each. Circle the shorter one.

💬 TALK ABOUT IT

The robot does exactly the same thing either way. So what is the repeat block actually saving?

What kind of path could a repeat block never shorten?

STUCK? TRY THIS

Look for the part of the path that happens again and again. Write that part once, then count how many times it happens.

★ CHALLENGE ZONE

Write a program with a repeat that draws a staircase going up to the right, three steps high.

A repeat can go inside another repeat. Write REPEAT 2 TIMES [REPEAT 3 TIMES [F] R] and work out what the robot draws.

WEEK 32 STEPS & RULES

Instructions With a Question

A program that looks at what is in front of it and decides.

IN CLASS THIS WEEK
Two-step word problems; reasoning (3.OA.8)

NUMBER WARM-UP · 2 MINUTES
If a number is bigger than 10, subtract 10. Otherwise leave it. Try it on 7 and on 14.

WORDS TO KEEP
IF … THEN … ELSE do one thing when the answer is yes, a different thing when it is no
condition the question the program asks

The robot now checks the square in front of it before it moves.

The program

REPEAT 6 TIMES [
IF the next square is shaded THEN R
ELSE F
]

1 Trace it.

a. On a row with **no** shaded squares, how far does the robot travel? _____

b. If **every** square is shaded, how far? _____

c. What is the robot doing in part b?

_____

2 A program with no robot in it.

Two cards, A and B. This program puts them in order.

IF A is bigger than B THEN swap them

ELSE do nothing

A	B	swap?	after
7	3		
2	9		
5	5		

Does the third row need a swap? _____ Is the program still right about it?

3 Both together.

Look back at Week 11. Sorting a hand of cards is a **repeat** with an **IF** inside it. Say in one sentence what the repeat does and what the IF decides.

_____

_____

✎ ADD TO YOUR COUNTING BOOK

Add a page called **Three kinds of instruction**: do this, do this again, and decide. Give one tiny example of each.

💬 TALK ABOUT IT

A program with an IF can behave differently on two different days. Is it still the same program?

What happens if the condition is never true? Is that a mistake?

STUCK? TRY THIS

Read the condition, answer it yes or no for the square you are actually on, and then follow only one of the two branches. Never both.

★ CHALLENGE ZONE

Write a program using a repeat and an IF that counts how many shaded squares are in a row of ten.

Change one word in the sorting program so it puts the cards in the wrong order, then say exactly what a person following it would do.

Counting Paths on a Grid

Every corner knows how many routes reach it, if you ask the two corners behind it.

IN CLASS THIS WEEK
Multiplication and addition patterns (3.OA.9)

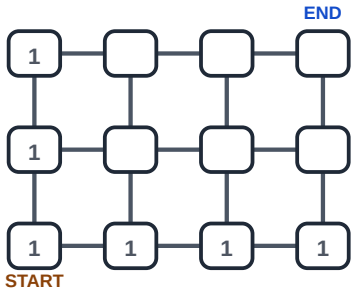
NUMBER WARM-UP · 2 MINUTES
What is 1 + 1? 1 + 2? 3 + 3? Keep those three answers in mind.

WORDS TO KEEP

route a way of walking from START to END going only right or up

build up work out the small answers first and let them make the big one

Streets, 3 across and 2 up. You may only walk **right** or **up**, never back. The boxes on the edges are filled in already.



1 Fill in the rest.

a. Why is every corner on the bottom edge a **1**?

_____

b. For any other corner, add the box **below** it and the box to its **left**. Fill them all in.

c. How many routes reach END? _____

2 More grids.

Grid	routes
2 across, 2 up	
3 across, 2 up	10
3 across, 3 up	
4 across, 2 up	

3 Why adding works.

To arrive at any corner you must have come from below it or from its left. There is no third way in. Explain why that means you add the two numbers.

✎ **ADD TO YOUR COUNTING BOOK**

Copy your filled-in 3 by 2 grid onto a page. Write beside it: **every box is the sum of the box below and the box to the left.** Next week those numbers show up somewhere else entirely.

💬 **TALK ABOUT IT**

Why can you never walk back to a corner you already left?

What would happen to the count if you were allowed to go down as well?

STUCK? TRY THIS

Work from the bottom left corner outward, not from END backward. Every box you need is already filled in by the time you get to it.

★ **CHALLENGE ZONE**

How many routes across a grid 4 across and 3 up?

Look at the numbers you wrote along the diagonal of the grid. Write them down in a row. You will meet them again in seven days.

The Triangle of Numbers

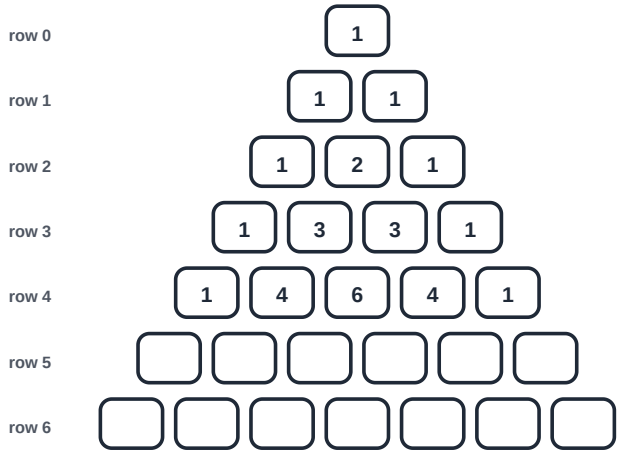
The route numbers, tipped over. And the handshakes are hiding in it.

IN CLASS THIS WEEK
Arithmetic patterns and explaining them (3.OA.9)

NUMBER WARM-UP · 2 MINUTES
1 + 4 = ? 4 + 6 = ? 6 + 4 = ?

WORDS TO KEEP
Pascal's triangle a triangle of numbers where each one is the sum of the two just above it. Named after Blaise Pascal, though it was known in China and India long before he was born

The first five rows are filled in. Every number is the sum of the two above it, and the edges are always 1.



1 Finish rows 5 and 6.

a. Row 5: 1, _____, _____, _____, _____, 1

b. Row 6: 1, _____, _____, _____, _____, _____, 1

2 Two things hiding in it.

a. Add up each row. Write the totals for rows 0 to 6.

b. Those totals are the answer to a question from Week 2. Which one?

c. Now read the **third** number in each row going down: 1, 3, 6, 10, 15. Where have you seen 15 before? _____

3 Check it against Week 8.

Six people at a party made 15 handshakes. Find 15 in the triangle and write which row it is in. Then work out which row 21 would be in, without drawing it.

✎ ADD TO YOUR COUNTING BOOK

Draw seven rows of the triangle on a full page. Mark the row sums down one side and circle the handshake numbers. This page is the best one in your Counting Book.

💬 TALK ABOUT IT

The routes on a grid, the ways to paint boxes red or blue, and the handshakes are three completely different questions. Why do they share one triangle?

What would row 7 be? Work it out before you look anywhere.

STUCK? TRY THIS

The two numbers above a box are the one up and left and the one up and right. On the edges there is only one number above, and the missing one counts as nothing.

★ CHALLENGE ZONE

Write out rows 7 and 8.

Find the number 20 in the triangle. Which grid from last week had 20 routes? That is not a coincidence.

WEEK 35 LOGIC & PROOF

Prove It With Pairing

Nine people, three handshakes each. It cannot happen, and here is why.

IN CLASS THIS WEEK
Even and odd; reasoning about arithmetic (3.OA.9)

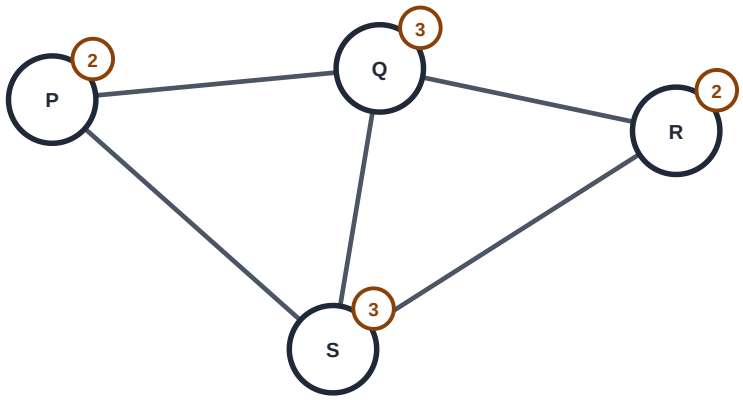
NUMBER WARM-UP · 2 MINUTES
Is 27 even or odd? Is 18 even or odd?

WORDS TO KEEP

proof a reason so solid that trying more examples would be a waste of time

must be even every handshake is counted twice, so the total of everybody's count can never be odd

Back to Week 9. Every line adds 1 to **two** dots, so the degrees always add up to twice the number of lines.



1 The nine people.

a. If 9 people each shake exactly 3 hands, what is the total of all their counts? 9 × 3 = _____

b. That total must be twice the number of handshakes. Is it even? _____

c. So how many handshakes would there be? Write what goes wrong.

_____

2 Possible or not?

People	each shakes	total	even?	possible?
9	3			
6	3			
7	3			
5	4			
11	5			

3 One row needs more care.

An even total means it is **not ruled out**. It does not promise a picture exists. Pick one of your possible rows and actually draw it.

✎ **ADD TO YOUR COUNTING BOOK**

Write the argument out properly: the claim, the reason, and the conclusion, in three sentences. This is the first real proof in your Counting Book, so give it its own page.

💬 **TALK ABOUT IT**

You never drew a single picture of 9 people and yet you are certain. Say why that is allowed.

Somebody says “I tried for an hour and could not do it, so it is impossible.” Is that a proof? What is the difference?

STUCK? TRY THIS

Count the total of everybody’s handshakes. Because every handshake gets counted once by each person in it, that total is always double something, so it can never be odd.

★ **CHALLENGE ZONE**

Show that in any room, the number of people who shook an **odd** number of hands is always even. Try a few rooms first, then say why.

Can 5 dots each have degree 3? You met this in Week 9 and could not finish it. Finish it now.

The Counting Book, Finished

Thirty-six weeks of pages. This week you turn them into one book.

IN CLASS THIS WEEK
Review of the year

NUMBER WARM-UP · 2 MINUTES
No warm-up. Get all your Counting Book pages out and put them in order.

Your Counting Book should have a page for nearly every week. Check it against this list and fix anything missing before you go on.

1 The check list

Tick each one you have.

Weeks	It should say	✓
1 to 2	list in a fixed order; the three questions	
3 to 5	the multiplication rule; arrays; trees	
6 to 8	order matters; order does not; the handshake rule	
9 to 12	degree; pigeonhole; sorting; halving	
14 to 18	area and perimeter; rectangles; factors	
19 to 24	primes; remainders; wrapping; scales	
26 to 28	equal parts; making one whole; equivalents	
29 to 34	coloring; matching; robots; routes; the triangle	
35	the proof	

2 One last count.

Use whichever tool fits. Name the tool as well as the answer.

a. A shop sells 4 flavors, 3 cone types and 2 toppings. How many different cones? _____

b. Ten friends, two go on a trip. How many pairs? _____

c. Today is Friday. What day in 50 days? _____

3 Teach one page.

Pick the page you are proudest of. Teach it to somebody who has never seen it, out loud, without reading it word for word. Write down the first question they asked you.

_____

✎ ADD TO YOUR COUNTING BOOK

Write a first page for the whole book: your name, the year, and one sentence saying what the book is for. Then number every page and make a contents list.

💬 TALK ABOUT IT

Which week changed how you count things the most?

Is there anything in the book you could not explain to somebody else? Go back to that page first.

STUCK? TRY THIS

A page you cannot explain out loud is not finished. That is the only test that matters for this week.

★ CHALLENGE ZONE

Look at the handshake rule, the routes on a grid, and Pascal's triangle. Write down what you think connects them. You will find out for certain in Grade 6.

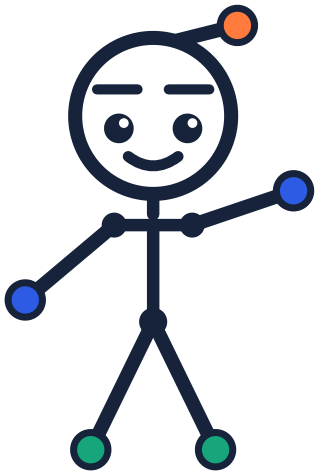
Invent one new counting puzzle of your own and put it on the last page of the book, with the answer written upside down at the bottom.

CERTIFICATE OF COMPLETION

Careful Counter

awarded to

for a whole year of listing in order, multiplying instead of counting, finding the factors, sorting the hand, coloring the schedule, and, hardest of all, proving that something *cannot be done*.



a Young Innovator for Change

Teacher

Date

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